

Exercises

The following exercises may be used.

1. The following table lists data points for the decay rate (in counts/s) of a radioactive source:

<i>Time</i> (s)	<i>Rate</i> (s ⁻¹)	<i>Time</i> (s)	<i>Rate</i> (s ⁻¹)	<i>Time</i> (s)	<i>Rate</i> (s ⁻¹)
0.6	18.4	2.0	3.02	3.6	1.72
0.8	10.6	2.4	2.61	4.0	1.61
1.2	8.04	2.8	2.08	4.2	1.57
1.6	6.10	3.0	1.50	4.3	1.85

- a. Plot the data using an appropriate set of axes, and determine over what range of times the rate obeys the decay law $R = R_0 e^{-t/\tau}$.
 - b. Estimate the value of R_0 from the plot.
 - c. Estimate the value of τ from the plot.
 - d. Estimate the value of the rate you expect at $t = 6$ s.
2. An experiment determines the gravitational acceleration g by measuring the period T of a pendulum. The pendulum has an adjustable

length L . These quantities are related as

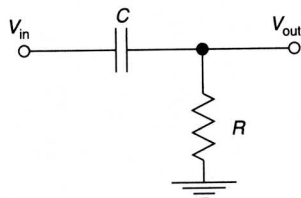
$$T = 2\pi \sqrt{\frac{L}{g}}.$$

A researcher measures the following data points in some arbitrary units.

Data point	L	T
1	0.6	1.4
2	1.5	1.9
3	2.0	2.6
4	2.6	2.9
5	3.5	3.4

One of these data points is obviously wrong. Which one?

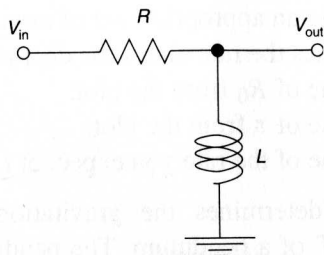
3. Consider the following simple circuit:



Let the input voltage V_{in} be a sinusoidally varying function with amplitude V_0 and angular frequency ω .

- Calculate the gain g and phase shift ϕ for the output voltage relative to the input voltage.
- Plot g and ϕ as a function of ω/ω_0 where $\omega_0 = 1/RC$. For each of these functions, use the combination of linear or logarithmic axes for g and for ϕ that you think are most appropriate.

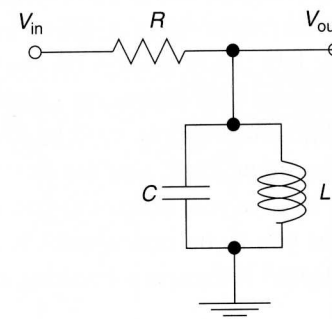
4. Consider the following simple circuit:



Let the input voltage V_{in} be a sinusoidally varying function with amplitude V_0 and angular frequency ω .

- Calculate the gain g and phase shift ϕ for the output voltage relative to the input voltage.
- Plot g and ϕ as a function of ω/ω_0 where $\omega_0 = R/L$. For each of these functions, use the combination of linear or logarithmic axes for g and for ϕ that you think are most appropriate.

5. Consider the following not-so-simple circuit:



- What is the gain g for very low frequencies ω ? What is the gain for very high frequencies? Remember that capacitors act like dead shorts and open circuits at high and low frequencies, respectively, and inductors behave in just the opposite way.
- At what frequency do you suppose the gain of this circuit is maximized?
- Using the rules for impedance and the generalized voltage divider, determine the gain $g(\omega)$ for this circuit and show that your answers to (a) and (b) are correct.

6. Suppose that you wish to detect a rapidly varying voltage signal. However, the signal is superimposed on a large DC voltage level that would damage your voltmeter if it were in contact with it. You would like to build a simple passive circuit that allows only the high-frequency signal to pass through.

- Sketch a circuit using only a resistor R and a capacitor C that would do the job for you. Indicate the points at which you measure the input and output voltages.

- b. Show that the magnitude of the output voltage equals the magnitude of the input voltage, multiplied by

$$\frac{1}{\sqrt{1 + 1/\omega^2 R^2 C^2}},$$

where ω is the (angular) frequency of the signal.

- c. Suppose that $R = 1 \text{ k}\Omega$ and the signal frequency is $1 \text{ MHz} = 10^6/\text{s}$. Suggest a value for the capacitor C .

7. An electromagnet is designed so that a 5-V potential difference drives 100 A through the coils. The magnet is an effective inductor with an inductance L of 10 mH. Your laboratory is short on space, so you put the DC power supply across the room with the power cables along the wall. You notice that the meter on the power supply must be set to 6 V in order to get 5 V at the magnet. On the other hand, you are nowhere near the limit of the supply, so it is happy to give you the power you need.

Is there any reason for you to be concerned? Where did that volt go, and what are the implications? If there is something to be concerned about, suggest a solution.

8. You are given a low-voltage, high-current power supply to use for an experiment. The manual switch on the power supply is broken. (The power supply is kind of old, and it looks like someone accidentally hit the switch with a hammer and broke it off.) You replace the switch with something you found around the lab, and it works the first time, but never again. When you take it apart, the contacts seem to be welded together, and you know it wasn't that way when you put it in. What happened? (*Hint: Recall that the voltage drop across an inductor is $L di/dt$, and assume the switch disconnects the circuit over 1 ms or so.*)

9. The following table is from the Tektronix Corp. 1994 catalog selection guide for some of their oscilloscopes:

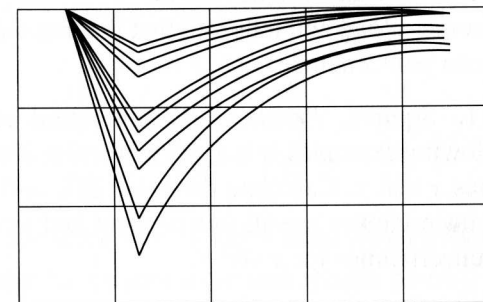
Model	Bandwidth	Sample rate	Resolution	Time bases
2232	100 MHz	100 MS/s	8 bits	Dual
2221A	100 MHz	100 MS/s	8 bits	Single
2212	60 MHz	20 MS/s	8 bits	Single
2201	20 MHz	10 MS/s	8 bits	Single

You are looking at the output of a waveform generator on one of these oscilloscopes. The generator is set to give a $\pm 2\text{-V}$ sine wave output. If the sine-wave period is set at $1 \mu\text{s}$, the scope indeed shows a 2-V amplitude.

However, if the period is 20 ns, the amplitude is 1 V. Assuming the oscilloscope is not broken, which one are you using?

10. You want to measure the energies of various photons emitted in a nuclear decay. The energies vary from 80 keV to 2.5 MeV, but you want to measure two particular lines that are separated by 1 keV. If you do this by digitizing the output of your energy detector, at least how many bits does your ADC need to have?

11. Pulses emitted randomly by a detector are studied on an oscilloscope: The vertical sensitivity is 100 mV/div and the sweep rate is 20 ns/div. The bandwidth of the scope is 400 MHz. The start of the sweep precedes the trigger point by 10 ns, and the input impedance is 50Ω .



- Estimate the pulse risetime. What could you say about the risetime if the bandwidth were 40 MHz?
- Estimate the trigger level.
- These pulses are fed into a charge-integrating ADC, also with 50Ω input impedance. The integration gate into the ADC is 100 ns long and precedes the pulses by 10 ns. Sketch the spectrum shape digitized by the ADC. Label the horizontal axis, assuming $\frac{1}{4} \text{ pC}$ of integrated charge corresponds to one channel.
- The ADC can digitize, be read out by the computer, and reset in 100 μs . Estimate the number of counts in the spectrum after 100 s if the average pulse rate is 1 kHz. What is the number of counts if the rate is 1 MHz?

12. A detector system measures the photon emission rate of a weak light source. The photons are emitted randomly. The system measures a rate of 10 kHz, but the associated electronics requires 10 μs to register a photon and the system will not respond during that time. What is the true rate at which the detector observes photons?

13. You measure the following voltages across some resistor with a three-digit DMM. As far as you know, nothing is changing so all the measurements are supposed to be of the same quantity V_R .

2.31	2.35	2.26	2.22	2.30
2.27	2.29	2.33	2.25	2.29

- Determine the best value of V_R from the mean of the measurements.
- What systematic uncertainty would you assign to the measurements?
- Assuming the fluctuations are random, determine the random uncertainty from the standard deviation.
- Somebody comes along and tells you that the true value of V_R is 2.23. What can you conclude?

14. (From G. L. Squires, *Practical Physics*, third ed., Cambridge (1985).) In the following examples, q is a given function of the independent measured quantities x and y . Calculate the value of q and its uncertainty δq , assuming the uncertainties are all independent and random, from the given values and uncertainties for x and y .

- $q = x^2$ for $x = 25 \pm 1$.
- $q = x - 2y$ for $x = 100 \pm 3$ and $y = 45 \pm 2$.
- $q = x \ln y$ for $x = 10.00 \pm 0.06$ and $y = 100 \pm 2$.
- $q = 1 - \frac{1}{x}$ for $x = 50 \pm 2$.

15. Police use radar guns to catch speeders. The guns measure the frequency f of radio waves reflected off of cars moving with speed v . This differs from the emitted frequency f_0 because of the Doppler effect

$$f = f_0 \left(1 - \frac{v}{c}\right)$$

for a car moving away at speed v . What fractional uncertainty must the radar guns achieve to measure a car's speed to 1 mph?

16. The period T of a pendulum is related to its length L by the relation

$$T = 2\pi \sqrt{\frac{L}{g}},$$

where g is the acceleration due to gravity. Suppose you are measuring g from the period and length of a particular pendulum. You have measured

the length of the pendulum to be 1.1325 ± 0.0014 m. You independently measure the period to within an uncertainty of 0.06%, that is, $\delta T/T = 6 \times 10^{-4}$. What is the fractional uncertainty (i.e., % uncertainty) in g , assuming that the uncertainties in L and T are independent and random?

17. You have a rod of some metal and you are changing its temperature T . A sensitive gauge measures the deviation of the rod from its nominal length $l = 1.500000$ m. Assuming the rod expands linearly with temperature, you want to determine the coefficient of linear expansion α , i.e., the change in length per Kelvin, and the actual length l_0 before any temperature change is applied. The measurements of the length deviation Δl as a function of the temperature change ΔT are as follows:

ΔT (K)	Δl (μm)	ΔT (K)	Δl (μm)	ΔT (K)	Δl (μm)
0.8	70	2.2	110	3.6	130
1.0	110	2.6	150	3.8	170
1.2	130	2.8	120	4.2	160
1.6	100	3.0	130	4.4	190
1.8	130	3.4	160	5.0	160

Plot the points and draw *three* straight lines through them:

- The line that best seems to go through the points.
- The line with the largest reasonable slope.
- The line with the smallest possible slope.

Use your own estimates by eye to determine these lines. (Do not use a fitting program.) Use the slopes and the intercepts of these lines to determine $\alpha \pm \delta\alpha$ and $l_0 \pm \delta l_0$.

18. For the previous problem, use the method of least squares to fit the data for Δl as a function of ΔT to a straight line. Use the fitted slope and the uncertainty to determine the coefficient of linear expansion α . Also calculate the uncertainty $\delta\alpha$. Are hand estimates just as good as a fitting program? What are the relative advantages or disadvantages?

19. Suppose you wish to measure the gravitational acceleration g by using something like the "Galileo" experiment. That is, you drop an object from some height h and you know that the distance it falls in a time t is given by $\frac{1}{2}gt^2$. For a given experimental run, the fractional uncertainty in h is $\delta h/h = 4\%$ and the fractional uncertainty in t is $\delta t/t = 1.5\%$. Find the fractional uncertainty in g from these data, assuming the uncertainties are random and uncorrelated.

20. You want to measure the value of an inductor L . First, you measure the voltage V across a resistor R when 1.21 ± 0.04 mA flows through it and

find $V = 2.53 \pm 0.08$ V. Then you measure the decay time τ in an RC circuit with this resistor and a capacitor C and get $\tau = RC = 0.463 \pm 0.006$ ms. Finally, you hook the capacitor up to the inductor and measure the oscillator frequency $\omega = 1/\sqrt{LC} = 136 \pm 9$ kHz. What is the value of L and its uncertainty?

21. A simple pendulum is used to measure the gravitational acceleration g . The period T of the pendulum is given by

$$T = 2\pi \sqrt{\frac{L}{g}} \left(1 + \frac{1}{4} \sin^2 \frac{\theta_0}{2} \right)$$

for a pendulum initially released from rest at an angle θ_0 . (Note that $T \rightarrow 2\pi \sqrt{L/g}$ as $\theta_0 \rightarrow 0$.) The pendulum length is $L = 87.2 \pm 0.6$ cm. The period is determined by measuring the total time for 100 (round trip) swings.

- A total time of 192 s is measured, but the clock cannot be read to better than ± 100 ms. What is the period and its uncertainty?
- Neglecting the effect of a finite value of θ_0 , determine g and its uncertainty from these data. Assume uncorrelated, random uncertainties.
- You are told that the pendulum is released from an angle less than 10° . What is the systematic uncertainty in g from this information?
- Which entity (the timing clock, the length measurement, or the unknown release angle) limits the precision of the measurement?

22. The β -decay asymmetry, A , of the neutron has been measured by Bopp *et al.* *Phys. Rev. Lett.* **56**, 919 (1986) who found that

$$A = \frac{2\lambda(1 - \lambda)}{1 + 3\lambda^2} = -0.1146 \pm 0.0019.$$

This value is perfectly consistent with, but more precise than, earlier results. The neutron lifetime, τ , has also been measured by several groups, and the results are not entirely consistent with each other. The lifetime is given by

$$\tau = \frac{5163.7 \text{ s}}{1 + 3\lambda^2}$$

and has been measured to be

918 ± 14 s by Christenson *et al.*, *Phys. Rev. D* **5**, 1628 (1972),

881 ± 8 s by Bondarenko *et al.*, *JETP Lett.* **28**, 303 (1978),
 937 ± 18 s by Byrne *et al.*, *Phys. Lett. B* **92**, 274 (1980), and
 887.6 ± 3.0 s by Mampe *et al.*, *Phys. Rev. Lett.* **63**, 593 (1989).

Which, if any, of the measurements of τ are *consistent* with the result for A ? Which, if any, of the measurements of τ are *inconsistent* with the result for A ? Explain your answers. A plot may help.

23. The “weighted average” of a set of numbers is

$$\bar{x}_W = \frac{\sum_{i=1}^n w_i x_i}{\sum_{i=1}^n w_i}, \quad (7.1)$$

where the “weights” $w_i \equiv 1/\sigma_i^2$.

- Prove that this definition for the weighted average is the value that minimizes χ^2 .
- Use propagation of errors to derive the uncertainty in the weighted average.

24. Let’s suppose you have some peculiar dice which each have 10 faces. The faces are numbered from 0 to 9. You throw *eight* of these dice at a time and record which numbers land face down on the table. You repeat this procedure (i.e., throwing the dice) 50 times.

- For how many throws do you expect there to be exactly three dice landing with either face 1 or face 5 landing face down?
- What is the average number of dice you expect to land with either face 1 or face 5 down, for any particular throw? What is the standard deviation uncertainty in this number?
- Use the Poisson approximation to calculate the same number as in (a).
- Use the Gaussian approximation to calculate the same number as in (a).

25. A radioactive source emits equally in all directions, so that the intensity falls off like $1/r^2$ where r is the distance to the source. You are equipped with a detector that counts only radioactivity from the source, and nothing else. At $r = 1$ m, the detector measures 100 counts in 10 s.

- What is the count rate, and its uncertainty, in counts per second?
- What do you expect for the *fractional* uncertainty in the count rate if you count for 100 s instead of 10?

- c. Based on the original 10-s measurement, predict the number of counts you should observe, and its uncertainty, if the detector is moved to a distance of 2 m and you count for 1 min.

26. Suppose you are using a Geiger counter to measure the decay rate of a radioactive source. With the source near the detector, you detect 100 counts in 25 s. To measure the background count rate, you take the source very far away and observe 25 counts in 25 s. Random counting uncertainties dominate.

- What is the count rate (in counts/s) and its uncertainty when the source is near the Geiger counter?
- What is the count rate (in counts/s) and its uncertainty when the source is far away?
- What is the net count rate (in counts/s) and its uncertainty due to the source alone?
- Suppose you want to reduce the uncertainties by a factor of 10. How long must you run the experiment?

27. An experimenter is trying to determine the value of “absolute zero” in degrees Celsius using a pressure bulb and a Celsius thermometer. She assumes that the pressure in the bulb is proportional to the *absolute* temperature. That is, the pressure is zero at absolute zero. She makes five measurements of the temperature at five different pressures:

Pressure (mm of Hg)	65	75	85	95	105
Temperature (°C)	-21	19	41	93	129

Use a straight line fit to determine the value of absolute zero, and its uncertainty, from these data.

28. Fit the following (x, y) values

$x =$	2.5	63	89	132	147
$y =$	406.6	507.2	551.3	625.5	651.7

to a straight line and plot the data points and the fitted line.

- Does it look like a straight line describes the data well?
 - Study this further by plotting the *deviations* of the fit from the data points. What does this plot suggest?
 - Try fitting the points to a quadratic form, i.e., a polynomial of degree 2. Is this fit significantly better than the straight line?
- 29.** The following results come from a study of the relationship between high school averages and the students' overall average at the end of the first

year of college. In each case, the first number of the pair is the high school average, and the second is the college average.

78.65	80.60	85.64	77.59
80.56	82.67	81.66	89.78
87.71	80.66	85.66	87.76
84.73	87.63	74.58	91.78
81.72	91.74	86.66	90.68

- Draw a scatterplot of the college average against the high school average.
- Evaluate the correlation coefficient. Would you conclude there is a strong correlation between the grades students get in high school and the grades they get in their first year of college?

30. Using the data in Table 2.1, draw a scatterplot of electrical conductivity versus thermal conductivity for various metals. (Electrical conductivity is the inverse of electrical resistivity.) Calculate the linear correlation coefficient.

31. Graph the ratio of the Poisson distribution to the Gaussian distribution for mean values $\mu = 2$ and for $\mu = 20$. Use this to discuss where the Gaussian approximation to the Poisson distribution is applicable. Repeat the exercise, but compare the Gaussian approximation directly to the binomial distribution with $p = \frac{1}{2}$.

32. Consider blackbody radiation.

- Show that the wavelength at which the intensity of a blackbody radiator is the greatest is given by “Wien’s displacement law”:

$$\lambda_{\max} \text{ (m)} = \frac{2.9 \times 10^{-3}}{T \text{ (K)}}.$$

Hint: You will need to solve an equation like $xe^x/(e^x - 1) = A$ for some value A . If $A \gg 1$ then this is trivial to solve, but you can be more exact using MATLAB. In MATLAB you would use the “function” `fzero` to find the place where $f(x) = A(e^x - 1) - xe^x$ crosses zero.

- Stars are essentially blackbody radiators. Our sun is a “yellow” star because its spectrum peaks in the yellow portion of the visible light. Estimate the surface temperature of the sun.

33. A particular transition in atomic neon emits a photon with wavelength $\lambda = 632.8 \text{ nm}$.

- Calculate the energy E of this photon.
- Calculate the frequency ν of this photon.
- An optical physicist tells you the “line width” of this transition is $\Delta\nu = 2$ GHz. What is the line width ΔE in terms of energy?
- Use the Heisenberg uncertainty principle to estimate the lifetime Δt of the state that emitted the photon.
- How far would a photon travel during this lifetime?
- Suppose the neon is contained in a narrow tube 50 cm long, with mirrors at each end to reflect the light back and forth and “trap” it in the tube. What is the nominal “mode number” for 632.8-nm photons, that is, the number of half-wavelengths that fit in the tube?
- What is the spacing in frequency between the nominal mode number m , and the wavelength corresponding to the mode $m + 1$?
- Compare the mode spacing $\delta\nu$ (part G) with the line width $\Delta\nu$.
- What is this problem describing?

34. Estimate the “transit time” for a typical photomultiplier tube. That is, how much time elapses between the photon ejecting an electron from the photocathode and the pulse emerging from the anode? Assume the photomultiplier has 10 stages and 2000 V between cathode and anode, divided equally among all stages, and that the dynodes are each separated by 1 cm.

35. Some high-quality photomultipliers can detect the signal from a single photoelectron, and cleanly separate it from the background noise. Such a PMT is located some distance away from a pulsed light source, so that on the average, the PMT detects $\langle N_{\text{PE}} \rangle$ photoelectrons. If $\langle N_{\text{PE}} \rangle \ll 1$ and N_0 pulses are delivered, show that the number of pulses detected by the photomultiplier is given by $\langle N_{\text{PE}} \rangle N_0$.

36. A photomultiplier tube observes a flash of green light from an Ar^+ laser. (Assume the photons have wavelength $\lambda = 500$ nm.) The photomultiplier is a 10-stage tube, with a RbCsSb photocathode. The voltages are set so that the first stage has a secondary emission factor $\delta_1 = 5$, while the other 9 stages each have $\delta = 2.5$. The laser delivers some huge number of photons to a diffusing system, which isotropically radiates the light, and only a small fraction of them randomly reach the photomultiplier. On the average, 250 photons impinge on the window for each flash of the laser.

- What is the average number of electrons delivered at the anode output of the photomultiplier tube, per laser flash?

- Assume these electrons come out in a rectangular pulse 20 ns wide. What is the height of the *voltage* pulse as measured across a 50- Ω resistor?
- You make a histogram of these pulse heights. What is the standard deviation of the distribution displayed in the histogram?
- Suppose the photomultiplier tube is moved four times farther away from the source. For any given pulse of the laser, what is the probability that no photons are detected?

37. A Geiger counter is a device that counts radioactive decays, typically used to find out whether something is radioactive. A particular Geiger counter measures 8.173 background counts per second; i.e., this is the rate when there are no known radioactive sources near it. Your lab partner hands you a piece of material and asks you whether it is radioactive. You place it next to the Geiger counter for 30 s and it registers a total of 253 counts.

- What do you tell your lab partner?
- What do you do next?

38. The Tortoise and the Hare have a signal-to-noise problem. A very weak signal sits on top of an enormous background. They are told to determine the signal rate with a fractional uncertainty of 25%, and they decide to solve the problem independently. The Tortoise dives into it and takes data with the setup, and he determines the answer after running the apparatus for a week. The Hare figures she is not only faster than the Tortoise but smarter too, so she spends two days reducing the background in the apparatus to *zero*, without affecting the signal. She then gets the answer after running the improved setup for one hour. (The Hare really is a lot smarter than the Tortoise, at least this time.)

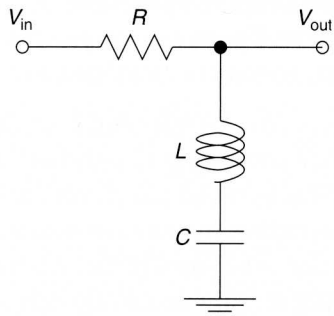
Assuming Poisson statistics,

- What is the signal rate?
- What is the Tortoise’s background rate?

39. Consider the passive filters shown in Fig. 3.11.

- Determine the gain as a function of $\omega = 2\pi\nu$ for each filter.
- Plot the gain as a function of ω/ω_C for the three low-pass filters. Define the critical frequency ω_C using the simplest combination of the two components in the circuit, that is, $\omega_C = 1/RC$, $\omega_C = 1/\sqrt{LC}$, or $\omega_C = R/L$. It is probably best to plot all three on the same set of log-log axes.

- c. Do the same as (b) for the high-pass filters.
 d. Can you identify relative advantages and disadvantages for the different combinations of low-pass and high-pass filters?
40. Consider the following variation on the circuit shown in Fig. 3.12:



- a. How does this circuit behave at high frequency?
 b. How does this circuit behave at low frequency?
 c. Calculate the gain $g = |V_{\text{out}}/V_{\text{in}}|$ as a function of frequency. What is the behavior for intermediate frequencies?
 d. Give an example of where this sort of filter would be useful.
41. A particle detector gives pulses that are 50 mV high when measured as a voltage drop across a 50- Ω resistor. The pulse rises and falls in a time span of 100 ns or less. Unfortunately, there are lots of noisy motors in the laboratory and the ground is not well isolated. The result is that a 10-mV, 60-Hz sine wave is also present across the resistor, and adds linearly with the pulses.
- a. Draw a simple circuit, including the 50- Ω resistor and a single capacitor, that allows the pulses to pass, but blocks out the 60-Hz noise.
 b. Determine a suitable capacitance value for the capacitor.
42. You are measuring a quantity Q that is proportional to some small voltage. In order to make the measurement, you amplify the voltage using a negative feedback amplifier, as discussed in Section 3.5.

- a. Show that the gain g of the full amplifier circuit can be written as

$$g = g_0 \left[1 - \frac{1}{\alpha\beta} + \mathcal{O}\left(\frac{1}{\alpha^2\beta^2}\right) \right],$$

where $g_0 = 1/\beta$ and $\alpha \gg 1$ is the internal amplifier gain, β is the feedback fraction, and $\alpha\beta \gg 1$.

- b. You measure Q with such an amplifier, with $\beta = 0.01$. The temperature in the lab fluctuates by 5°F while you make the measurement, and the specification sheet for the opamp tells you that its gain varies between 2.2×10^4 and 2.7×10^4 over this temperature range. What is the fractional uncertainty in Q due to this temperature fluctuation?

43. A ^{22}Na radioactive source emits 0.511- and 1.27-MeV γ -rays. You have a detector placed some distance away. You observe a rate of 0.511-MeV photons to be $2.5 \times 10^3/\text{s}$, and of 1.27-MeV photons to be $10^3/\text{s}$, with just air between the source and the detector. Calculate the rate you expect for each γ -ray if a 1/2-in.-thick piece of iron is placed between the source and the detector. Repeat the calculation for a 2-in.-thick lead brick.

44. A radioactive source is situated near a particle detector. The detector counts at a rate of $10^4/\text{s}$, completely dominated by the source. A 2-cm-thick slab of aluminum (density 2.7 gm/cm³) is then placed between the source and the detector. The radiation from the source must pass through the slab to be detected.

- a. Assuming the source emits only 1-MeV photons, estimate the count rate after the slab is inserted.
 b. Assuming the source emits only 1-MeV electrons, estimate the count rate after the slab is inserted.

45. Consider a small rectangular surface far away from a source. The surface is normal to the direction to the source, and subtends an angle α horizontally and β vertically. Show that the solid angle subtended is given by $\alpha\beta$.

46. A photomultiplier tube with a 2-in. active diameter photocathode is located 1 m away from a blue light source. The face of the PMT is normal to the direction of light. The light source isotropically emits 10^5 photons/s. Assuming a quantum efficiency of 20%, what is the count rate observed by the photomultiplier?

47. Two scintillation detectors separated by 3 m can measure the “time-of-flight” for a particle crossing both of them to a precision of ± 0.20 ns. Each detector can also measure the differential energy loss $dE/dx = \text{constant}/\beta^2$, $\beta = v/c$, to $\pm 10\%$. For a particle with a velocity of 80% the speed of light (i.e., $\beta = 0.8$), how many individual detectors are needed

along the particle path to determine the velocity v using dE/dx to the same precision as is possible with time-of-flight?

48. A Čerenkov detector is sensitive to particles that move faster than the speed of light in some medium, i.e., particles with $\beta > 1/n$, where n is the index of refraction of the medium. When a particle crosses such a detector, it produces an average number of detected photons given by

$$\mu = K \left(1 - \frac{1}{\beta^2 n^2} \right).$$

The actual number of detected photons for any particular event obeys a Poisson distribution, so the probability of detecting no photons when the mean is μ is given by $e^{-\mu}$. When 1-GeV electrons ($\beta = 1$) pass through the detector, no photons are observed for 31 out of 19,761 events. When 523-MeV/c pions ($\beta = 0.9662$) pass through, no photons are observed for 646 out of 4944 events. What is the best value of the index of refraction n as determined from these data? What is peculiar about this value? (You might want to look up the indices of refraction of various solids, liquids, and gases.)

- Absorption coefficient, 300–301
- AC circuits, 93–96
- ADC. *See* Analog-to-digital converter
- Airy equation, 190
- Alpha particles, 306, 324, 325f, 351–354
- Amplifiers, operational, 119–120
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