

Rice University  
Physics 332

## RUTHERFORD SCATTERING

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## I. Introduction

The most popular early model of the atom was proposed by J. J. Thomson in 1904. He suggested that atoms consisted of electrons embedded in a uniform positive charge distribution, rather like raisins in cake. Since the electrons were thought to comprise most of the mass of the atom, they had to be very numerous. The contrast with the modern picture of a relatively small number of electrons surrounding a tiny, massive nucleus is quite sharp.

In 1910 Thomson calculated the way in which his atoms would deflect rapidly-moving charged particles. Initial experiments with both electrons and alpha particles seemed to confirm Thomson's picture, but the work raised questions in Ernest Rutherford's laboratory. A more detailed study was therefore proposed by Rutherford and carried out by Hans Geiger with assistance from Ernest Marsden. (At the start of their joint work Marsden was an undergraduate.) They set out to measure the number of alpha particles scattered out of a collimated beam upon hitting a thin metal foil. They determined the angular distribution of the scattered particles for several different materials, thicknesses and alpha energies. To their initial surprise, Geiger and Marsden found that some alpha particles were scattered through large angles in atomic collisions. This amount of deflection was problematic in Thomson's model, and eventually led Rutherford to suggest his alternative model, in which a small number of electrons orbit a massive, point-like positive nucleus.

A number of facts about the alpha particle were already known when Geiger and Marsden undertook their experiments. It was emitted in the radioactive decay of radium and several of its daughters. Each material emitted alphas of characteristic energy, and the energy could be subsequently reduced by passing the particles through metal foils. The particle itself was known to have approximately the mass of a helium atom, and to carry two units of positive charge, suggesting that it was a helium atom that had lost two electrons. Whether this meant that the particle was still the size of an atom, as in the Thomson model, or much smaller than an atom, as in the Rutherford model, was unclear.

In this experiment you will have the opportunity to reproduce, with modern techniques, some of Geiger and Marsden's measurements. To show the logic behind the experiment, we will first define the scattering cross-section  $\sigma$ , and then briefly present the scattering calculations for the Rutherford atom. The remaining sections discuss the measurement techniques and analysis.

## II. Theoretical Considerations

### A. General scattering theory

The general geometry of a scattering experiment is shown in Fig. 1. A parallel beam of particles is incident on a target, which deflects some of the particles through an angle  $\theta$  relative to the original beam direction. The particles are counted when they hit the detector plane. For the example shown, only the annulus between  $r_{in}$  and  $r_{out}$  is sensitive, so particles must fall within a restricted range of angles to be counted.

It is customary to describe scattering experiments in terms of the differential cross-section  $d\sigma/d\Omega$ . If  $N$  incident particles strike a foil of thickness  $t$  containing  $n$  scattering centers per unit volume, the average number  $dN$  of particles scattered into the solid angle  $d\Omega$  around  $\Omega$  is given by

$$dN = Nnt \frac{d\sigma}{d\Omega} d\Omega \quad (1)$$

In general terms,  $d\sigma$  can be thought of as the effective area surrounding the scattering center which the incident particle must hit in order to be scattered into  $d\Omega$ . This is easy to understand if one sees that the total "target area" is simply  $nt d\sigma$  and that the average number of particles striking this area is then  $Nnt d\sigma$ . All  $dN$  of these particles emerge in the angular range  $d\Omega$  near  $\Omega$ ,

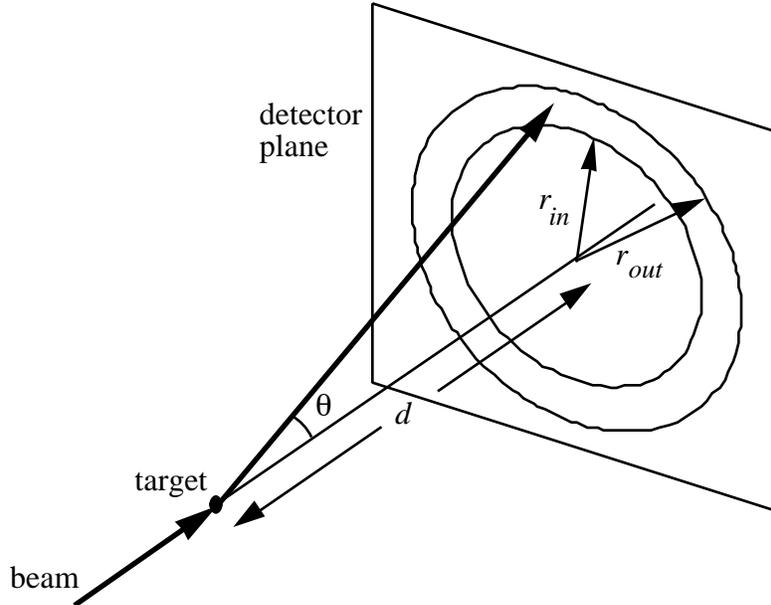


Fig. 1 Schematic of a scattering experiment. The detector is assumed to be an annular ring of radii  $r_{in}$  and  $r_{out}$  placed a distance  $d$  beyond the target.

by assumption.

In an actual experiment, one illuminates a target of known  $t$  and  $n$  with a beam of particles and counts the number of those particles  $\Delta N$  scattered into a finite solid angle  $\Delta\Omega$  centered at  $\Omega$ . It is convenient to count for a known time  $\Delta T$  and then express the scattering in terms of rates

$$\frac{\Delta N}{\Delta T \Delta\Omega} = \frac{N}{\Delta T} n t \frac{d\sigma}{d\Omega} \quad (2)$$

where  $N/\Delta T$  is the beam rate during the measurement. Provided  $\Delta\Omega$  is small, Eq. 2 shows how the measured ratio  $\Delta N/\Delta T \Delta\Omega$  can be compared with the cross section deduced from a theoretical model.

To calculate the solid angle it is convenient to introduce spherical polar coordinates centered at the point where the incident beam strikes the target, with the polar axis along the incident beam direction. Then

$$\Delta\Omega = \int_{\text{det}} d\Omega = \int_{\text{det}} \sin\theta \, d\theta d\phi \quad (3)$$

where the integration runs over the part of the detector seen by the target. It frequently happens, as suggested by Fig. 1, that there is no  $\phi$  dependence and Eq. 2 and 3 can be simplified accordingly.

## B. Scattering from a Rutherford atom

Evidently a head-on collision with the heavy nucleus of a Rutherford atom could drastically alter the path of the much lighter alpha particle. Conversely, much of the atomic volume was now assumed to contain only the very light electrons, which could not produce much deflection. Accordingly, many alpha particles would pass through the atom, deflected only slightly by the electrons and the long-range Coulomb interaction with the nucleus.

Rutherford's calculation of the cross-section for his model is now presented in most mechanics texts, so only the result is given here:

$$\frac{d\sigma}{d\Omega} = \frac{q^2}{16} \sin^{-4}(\theta/2) \quad (4)$$

where

$$q = 2Ze^2 / E, \quad (5)$$

$Ze$  is the charge on the target atom,  $2e$  is the charge and  $E$  the energy of the alpha particle, all in CGS units. (Incidentally, a full quantum calculation gives exactly the same answer.) The probability of scattering falls off quickly with angle, which makes an experimental test difficult.

In deriving this expression Rutherford assumed that the Coulomb potential of the nucleus extended out indefinitely, so that all the alphas are deflected at least somewhat. (This is the reason the cross section diverges when integrated over all angles.) For a real material, this effect is substantially reduced because the atomic electrons act to screen the nuclear potential from alphas that pass relatively far from the nucleus. At the same time, numerous collisions with the electrons tend to deflect the alphas, thereby increasing the average deflection on traversing the target. Together these effects are called multiple scattering, because the net deflection of the projectile is the result of many small-angle, random scattering events. A proper treatment of the multiple scattering is obviously difficult and messy, so one tries to minimize the error by using thin targets and limiting the analysis to angles beyond the likely range of multiple scattering effects. A more rigorous test would exploit the fact that the Rutherford scattering is proportional to  $t$ , whereas the multiple scattering should be proportional to  $t^{1/2}$

### III. Experimental Methods

The objective of this experiment is to see if the angular distribution of scattered alpha particles follows Eq. 4. To do this we need a beam of particles, the scattering foil and an alpha particle detector. Since the range of alpha particles in air is only about a centimeter, the apparatus must be enclosed in a vacuum chamber. We briefly describe each of these components and then provide a detailed procedure for doing the measurements.

#### A. Equipment

An overall sketch of the equipment is shown in Fig. 2, and photos are available in the lab. The source is one millicurie of  $^{244}\text{Cm}$ , which emits 5.8 MeV alphas. About 1 MeV is lost in passing through the titanium cover foil which prevents the escape of radioactive material. The emitted alphas are formed into a beam by an aperture a few centimeters from the source. After leaving the collimator, the beam strikes a gold foil mounted over a hole in an aluminum flap. The flap can be moved to put the gold into the beam, or to let the beam pass through an identical hole with no target foil. All of these parts are mounted on a movable carriage which can be placed at a known distance from the detector by means of a screw drive.

The detector is a thin layer of ZnS:Ag powder glued to a plexiglass window in the vacuum chamber wall. Alpha particles hitting the ZnS:Ag produce flashes of light which are detected by a photomultiplier tube and then counted in a pulse height analyzer. The sensitive area of the detector is determined by a mask with an annular opening. This allows particles scattered into a range of  $\theta$ , determined by the target to detector distance and mask radii, to reach the scintillator. There is also a small hole in the center of the mask, normally covered by a shutter. With the

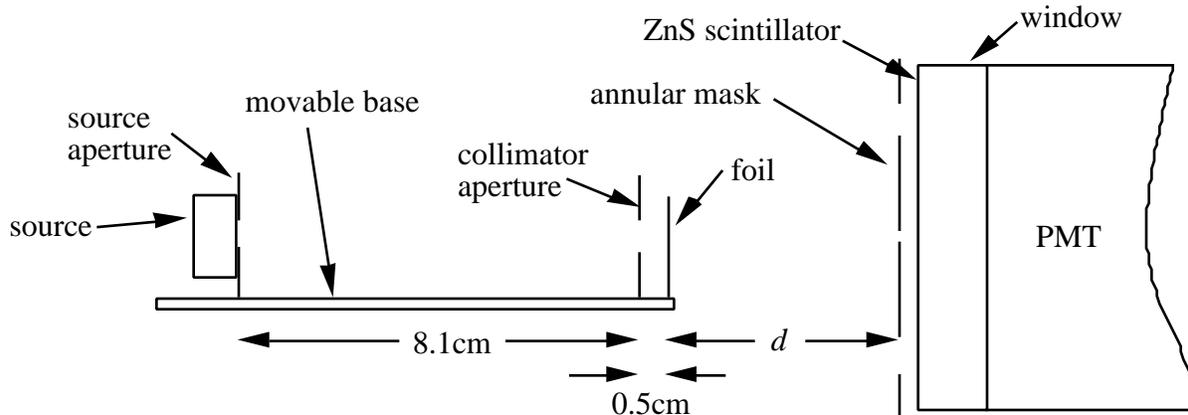


Fig. 2 Schematic of the apparatus. The source, collimator and target are all attached to a movable platform. The distance  $d$  is adjustable by means of a screw drive (not shown). A mask covers the scintillator, allowing particles to hit the active surface only within an annulus.

shutter open, the mask can be scanned laterally to align it on the axis of the beam.

## B. Measurements

To take data one must get signals to the counting electronics, align the mask with the beam, and count the number of scattered events at several positions of the movable carriage corresponding to a range of  $\theta$ . Counts must be made with the target both in and out of the beam so that scattering from the target can be distinguished from extraneous events.

Before proceeding, the apparatus must be evacuated. To protect the very fragile foil this must be done carefully, so consult the instructor if the chamber is not already under vacuum. Please do not attempt to pump it out on your own, and do not turn off the vacuum pump.

Connect the photo tube to the amplifying electronics as indicated in Fig. 3. The output of the amplifier should go to the PHA input of the computerized analyzer. Start the PCA software and configure it for PHA mode, following the instructions in the local guide. With the high voltage at +1400V, the shutter open, and the target out you should see events appearing on the PHA screen. The rate will be best if the movable carriage is fairly close to the detector, say at about 10 turns. Adjust the amplifier gain so that the largest pulses fall somewhat below full scale.

Because our scintillator is a powder that scatters the scintillation light, the pulse spectrum is not very informative, but we can still use the PHA to minimize the effect of electronic noise. Follow the instructions in the PCA operating manual to set a region of interest, ROI, from just above the low-level noise peak to the last channel. If the cursor is somewhere in the ROI and you are not actually counting, the computer will display the total counts in this region, most of which should be due to alpha impacts. These readings are our basic data.

You can now locate the center of the beam spot. Leave the movable carriage fairly close to the detector and count at enough lateral positions of the mask to determine the shape of the incoming beam. Plot the rate vs mask position, and determine the setting that puts the center of the mask on the center of the beam spot.

The screw drive on the source-target carriage has a range from about 0-12 turns. Using the calibration equation and mask radii given in the appendix you can translate a setting in turns to a

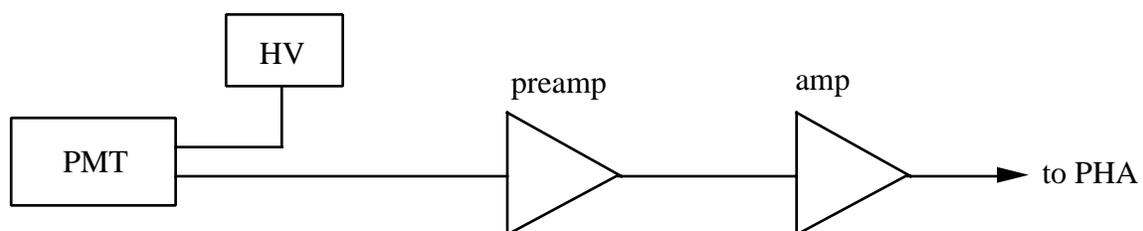


Fig. 3. Block diagram of electronics. The high voltage should be set at +1400 V. The preamp is an Ortec model 113, set for 100 pF input capacitance. The amplifier is an Ortec model 485 set for negative input, bipolar output. Adjust the gain as described in the text.

scattering angle  $\theta$  and subtended solid angle  $\Delta\Omega$ . Plan your settings so that you get several points in the available range, more or less evenly spaced in  $\sin^4(\theta/2)$ . You will also need to pay attention to counting times so that all of your data are sufficiently accurate despite the large variation in counting rate.

When you have decided on a strategy, set the position as needed and count for the appropriate time with the target in. The raw data are the total counts in the ROI, but it is probably good practice to save the whole spectrum to disk in case you want to try changing the ROI later. You will also need to repeat the count at each position with the target out. This reading should be subtracted from the target-in count to compensate for cosmic ray counts and for alphas that scatter somewhere else in the apparatus but still reach the detector.

### **C. Analysis**

To analyze the data, compute the solid angle  $\Delta\Omega$  at each of the scattering angles you chose, and plot  $\Delta N/\Delta T\Delta\Omega$ . A log-log plot of  $\Delta N/\Delta T\Delta\Omega$  vs  $\sin^4(\theta/2)$  is a compact way to present the results. For comparison you should also calculate the expected rate from Eqs. 2 and 4, and put it on the same plot. Try to decide if your data generally agree with the Rutherford model, and discuss any discrepancies that you notice.

## Appendix

### Some useful constants

electron charge $e$	$4.80 \times 10^{-10}$ esu
alpha energy $E$	4.4 MeV (average in foil)
charge on gold $Z$	79
half-life $^{244}\text{Cm}$	18.10 years

### Some apparatus parameters

beam rate (14 Mar 1995)	$1.50 \times 10^3$ particles/s
source aperture	0.4 cm
collimator aperture	0.2 cm
source to collimator	8.1 cm
collimator to target	0.5 cm
area density of foil $nt$	$3.4 \times 10^{18}$ cm <sup>-2</sup>
target to detector $d$	-1.268 cm/turn x turns + 16.11 cm
mask radius $r_{in}$	2.455 cm
mask radius $r_{out}$	3.040 cm