

Geiger Counter, Statistics, and Lifetime Determination

References

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I. Geiger Counter

A Geiger counter is a charged capacitor with the region between the electrodes filled with a gas (Fig. 1). As ionizing radiation pass through this gas, the molecules of the gas are ionized, so that the gas contains free electrons and positive ions. The electric field in the capacitor separates the ions from the free elections and prevents them from recombining.

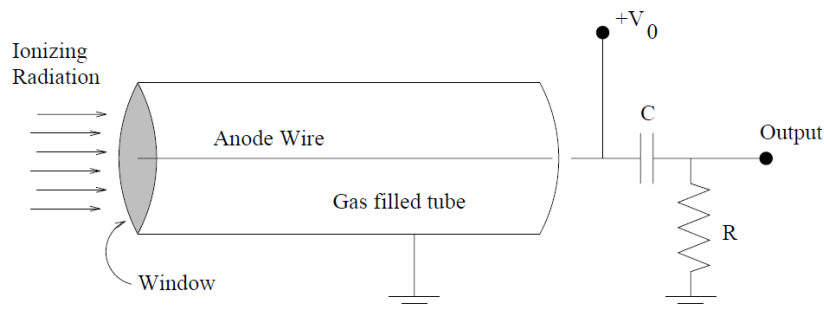


Fig. 1. In a Geiger counter the anode wire is raised to a high voltage V_0 (through a very high resistance, not shown). An avalanche of charge in the tube discharges capacitor C through load resistor R , creating a voltage pulse on the output.

If air is used as the counter gas, then the energy needed to form an ion-electron pair is about 34 eV, so a 1 MeV gamma ray will produce about 3×10^4 such pairs, and with a typical capacitance of 10^{-11} farads, one will observe a voltage of about $\frac{1}{2}$ millivolt, which is too small to be detected in our apparatus. The counter, when operating in this mode, is called an *ionization chamber*; you will not be able to observe counter operation in this mode.

As the voltage on the counter is increased, the electrons that are released through ionization of the counter gas by the original ionizing radiation are accelerated. When the voltage

is large enough, these accelerated electrons can themselves ionize the counter gas, giving rise to more ion-electron pairs. Thus the current in the capacitor that forms the counter is amplified through the production of these secondary electrons. This is a *Townsend avalanche*. The current through the counter is found to be proportional to the applied voltage. This device is called a *proportional counter*.

If the voltage is increased still further, photons generated in the ionization process can participate in ionizing the counter gas, and these photons can travel throughout the counter. So, when this point is reached, the entire counter is participating in delivering current and the counter becomes *saturated*. The current through the counter becomes independent of the voltage, and the device is called a *Geiger-Müller counter*.

The output signal consists of the collected electrons from the avalanche processes. The collection time is of the order of 10^{-6} sec, during which time the positive ions do not move very far. This process therefore produces a cloud of positive ions around the anode which reduces the electric field and eventually terminates the avalanche process. The result is a pulse of current from all the collected electron-ion pairs.

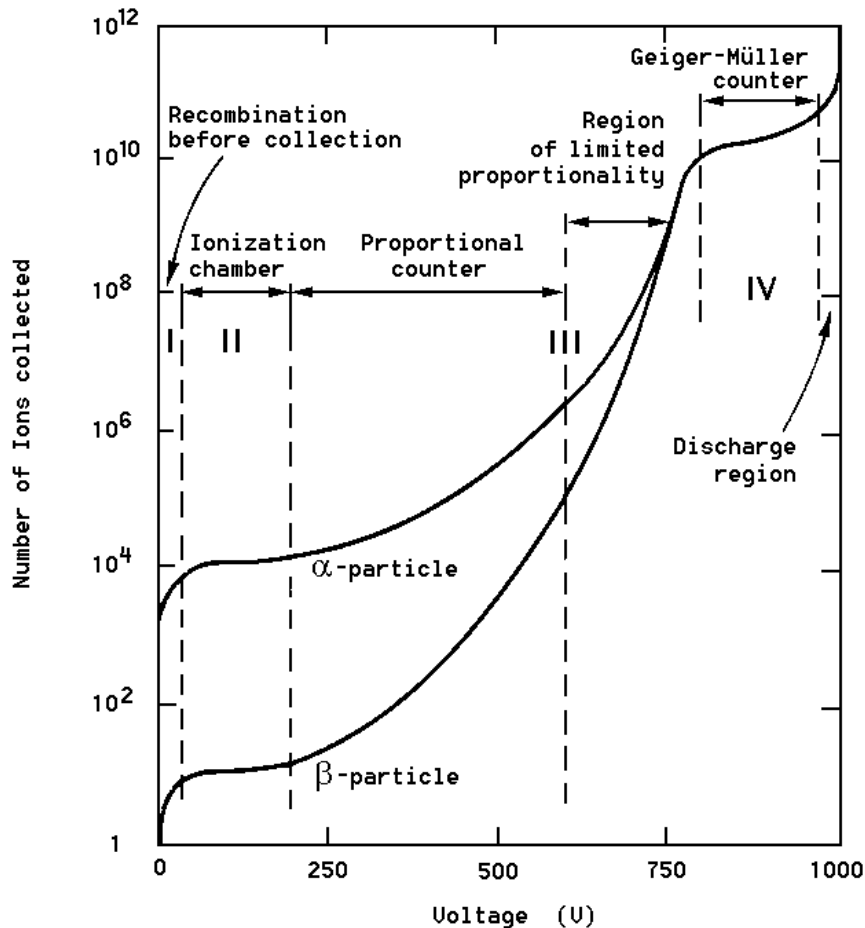


Fig. 2. Number of electron-ion pairs collected (thus also pulse height) vs voltage for gas-filled counters. Count rate vs voltage also exhibits the same behavior.

In order to help terminate the avalanches, a quenching gas is added to the counter gas

during manufacture. The quenching gas usually consists of organic molecules such as ethanol, while the counter gas might be argon or nitrogen. Dissociation of the organic molecules can prevent ionization by accelerated ions, which would otherwise cause the release of new electrons and resumption of the avalanche process. When the quenching gas is all dissociated, the Geiger tube fails and must be replaced.

There is a considerable voltage region where the counter current in a pulse is independent of voltage (Fig 2). This *plateau* region is where one should operate a Geiger counter. If one raises the voltage too high, successive Geiger pulses are generated by a single ionizing event, and an audible ticking noise can be heard. At this point, the quenching gas is being rapidly destroyed, and unless the voltage is reduced, the counter will fail. IF TICKING IS HEARD, REDUCE THE VOLTAGE AND CALL YOUR INSTRUCTOR OR TA.

For the counters being used in the laboratory, ONE MUST NOT RAISE THE VOLTAGE ABOVE 1000 VOLTS (1200 VOLTS FOR SOME OF THE TUBES).

The current in the counter is converted to a voltage by passing it through coupling capacitor and a load resistor (Fig. 1). The voltage pulse is then passed through a discriminator, which selects for counting those pulses larger than a specific voltage and rejects the others. At fixed threshold voltage, the count rate from a constant source of ionizing radiation becomes independent of voltage in the plateau. Thus in the proportional region, the number of counts per second recorded by the scalar is proportional to the GM tube voltage, while in the Geiger region, it is independent of the voltage.

In this part of the experiment find the count rate as a function of counter voltage, using a ¹³⁷Cs source. First read the APPARATUS AND SAFETY NOTES at the end of this Guide. Before turning on the high voltage switch, be sure the voltage knob is set fully counter-clockwise to zero volts. After you turn on the counter, turn on the high voltage to the GM tube and gradually increase the voltage from zero. Start at the voltage where the counter just starts to count (the *threshold*) and proceed up to 300 volts above threshold, BUT IN NO CASE ABOVE 1000 [1200] VOLTS (25 volt increments. are a good choice). Make sure you get through the plateau and just begin to see the count rate rise. Plot your count rate vs voltage and determine where the proportional and Geiger regions are. As will be seen below, the error assigned to an observation of n counts is $\pm\sqrt{n}$. Error bars showing this error level should appear on your plot. Keep complete notes in your lab notebook.

II. Counting Statistics: The Binomial Distribution

All laboratory measurements contain sources of uncertainty or error. Some originate with the properties of the measuring instrument (such as trying to estimate fractions of a millimeter on a meter stick graduated in millimeters). Others, of which radioactive decay is one, originate with the inherent statistical variations of a process whose occurrence is essentially random. If we make a single measurement of a phenomenon governed by a random, statistical process, then the outcome of the measurement is useful to us only if we can provide the answers to two questions: (1) How well does the measurement predict the

outcome of future measurements? (2) How close to the "true" value is the outcome of a single measurement likely to be? To answer these questions, we must know how the various possible outcomes are distributed statistically.

Let us suppose we have a sample of M nuclei, and we wish to compute the probability $P(n)$ that n of them decay in a certain time interval. (For the moment we assume that we can measure time with arbitrarily small uncertainty.)

The probability for decay of a single nucleus in this time interval is p . Of course for your measurements p includes all the geometry and detection effects of your apparatus. We assume that p is constant – each nucleus decays independently of the state of the other nuclei. The desired probability can be found from the *binomial distribution*

$$P(n) = \frac{M!}{n!(M-n)!} p^n (1-p)^{M-n} \quad (1)$$

Note that $M!/ [n!(M-n)!]$ is the number of combinations of M things taken n at a time, that p^n is the probability for a particular set of n nuclei to decay, and that $(1-p)^{M-n}$ is the probability for the remaining $M-n$ nuclei not to decay. In your lab notebook explain each of these three factors and why you would measure only the product.

The binomial distribution is most often encountered in simple random experiments, such as tossing a coin or rolling dice. The distribution is characterized by its mean \bar{n} , which is equal to pM (as one would guess: the decay probability per nucleus times the total number of nuclei gives the number of decays) and also by the variance σ^2 . These two quantities are defined as follows:

Consider what happens when you make repeated measurements of the number of counts in some fixed time interval. If the data consist of N measured values of n , call them n_1, n_2, n_3, \dots , then the mean \bar{n} and standard deviation σ , are given by

$$\bar{n} = \frac{1}{N} \sum_{i=1}^N n_i \quad (2)$$

$$\sigma^2 = \frac{1}{N-1} \sum_{i=1}^N (n_i - \bar{n})^2 \quad (3)$$

The standard deviation σ which is the square root of the variance, is a rough measure of the "width" of the statistical distribution. For the binomial distribution, $\bar{n} \rightarrow pM$ and $\sigma^2 \rightarrow \bar{n}(1-p)$.

III. Determination of Background: The Poisson Distribution

When n and M are small, the binomial distribution is quite trivial to use, but when n and M are very large, as might be typical in decay processes, the distribution becomes less useful as the computations become difficult. We can obtain a less cumbersome approximation in the case when $p \ll 1$ (which will usually be true for radioactive decays):

$$P(n) = \frac{\left(\bar{n}\right)^n e^{-\bar{n}}}{n!} \quad (4)$$

where $\bar{n} = pM$ as before. This is known as the Poisson distribution. Note that the probability of observing n decays depends only on the mean value \bar{n} . For this distribution,

$$\sigma = \sqrt{\bar{n}} \quad (5)$$

The Poisson distribution can be derived from the binomial distribution in the case that $p \ll 1$.

In this experiment, you are going to first analyze the background radiation counts in a Geiger counter. That is, the counter is turned on with no radioactive source present. Then the number of counts occurring in a suitably chosen time interval is observed. This observation is repeated N times, where N is a large number. The number of times n counts is observed is the frequency, $f(n)$. Note that

$$\sum_n f_n = N \quad (6)$$

Therefore, to obtain an estimate of the probability of observing n counts in the time interval chosen, P_n , one multiplies the frequency by a factor $1/N$; conversely, one may form, a frequency function, $f(n)$, by multiplying $P(n)$ by N :

$$f(n) = NP(n) \quad (7)$$

Observing a Poisson distribution and determining the background in your counter

The background counts are always present, so you will have to subtract the mean background counts from your measurements of counts from radioactive samples. So first you must measure the background. The way to do this measurement is to keep the voltage set at the previously chosen operating point and remove any radioactive source from the tube holder. Then, choose a time interval which will give an average of 2-3 background counts per time interval. Now record the *actual* number of counts in $N = 50$ such intervals.

Make a *histogram* of your data -- a sort of bar chart with number of counts per interval, n , as the horizontal axis and number of times a particular count appeared (frequency, f_n) as

the vertical axis, as shown in Fig. 3. (A more familiar histogram is a class grade distribution with n representing a particular score and f_n the number of students receiving that score).

Find the *sample* mean of your data by dividing the total number of counts by the number of trials. Use this as an *estimate* of \bar{n} in the formula for the Poisson distribution.

Plot the Poisson distribution $f(n)$ on the same histogram to compare theory with experiment.

The measured and predicted frequencies are different, of course. If one repeated this experiment many times, the mean of the measured frequencies f_n for any particular value of n , should converge to the value predicted by the Poisson distribution $f(n)$. The standard deviation of these frequencies should be $\sqrt{f(n)}$. Thus your data can be taken as represented by a Poisson distribution provided 95% of it is within two standard deviations of the Poisson values. Is this the case? Plot and discuss in your lab book.

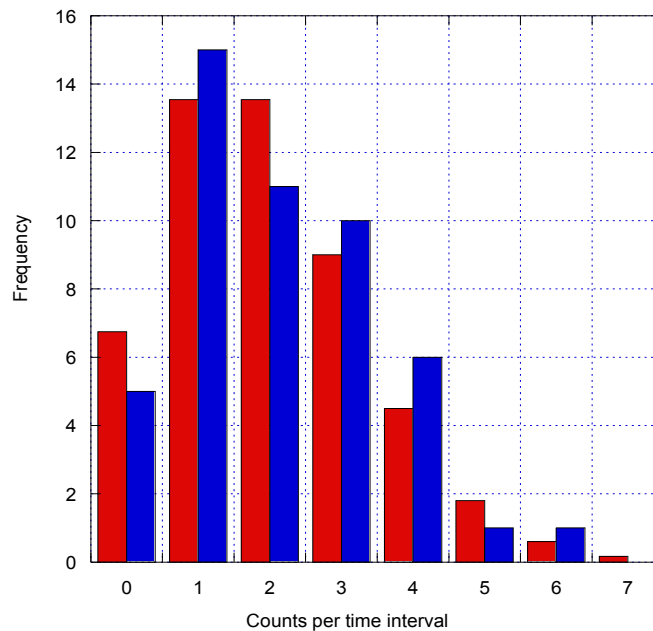


Fig. 3. Sample histograms of background counts (blue) and matching Poisson distribution (red)

IV. Radioactive Source: Gaussian Distribution

Returning to the discussion of the binomial distribution, another useful approximation occurs in the case of small p and large \bar{n} , in which case the “normal” or *Gaussian* distribution is obtained:

$$P(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-(n - \bar{n})^2 / 2\sigma^2\right] \quad (8)$$

and once again, $\sigma = \sqrt{\bar{n}}$. (9)

The Poisson distribution becomes identical to the Gaussian distribution when the number of data points becomes large, and the Gaussian distribution has the standard deviation given above (Fig. 4). The Gaussian distribution has the property that 68% of the probability lies within $\pm 2\sigma$ of the mean value \bar{n} . Unfortunately the mean value \bar{n} is not available for measurement; it results formally from an infinite number of trials. Clearly the "true" value we are seeking is represented by \bar{n} , and our single measurement n has a 68% chance of falling within $\pm\sigma$ of \bar{n} , we therefore take n as the best guess for \bar{n} and we quote the error limit on our estimate n as $\pm\sigma$, or $\pm\sqrt{n}$.

In this experiment, you will use a ^{137}Cs source to investigate a situation that has approximately a Gaussian distribution. The numbers, n , will be on the order of 1000. Thus the standard deviation will be about 30, and 95% of the data will lie between $1000-60$ and $1000+60$. There would be about 120 values of n to contend with, unless some grouping of data were done. In fact, by putting data in bins of width Δn , one may make the data analysis more tractable.

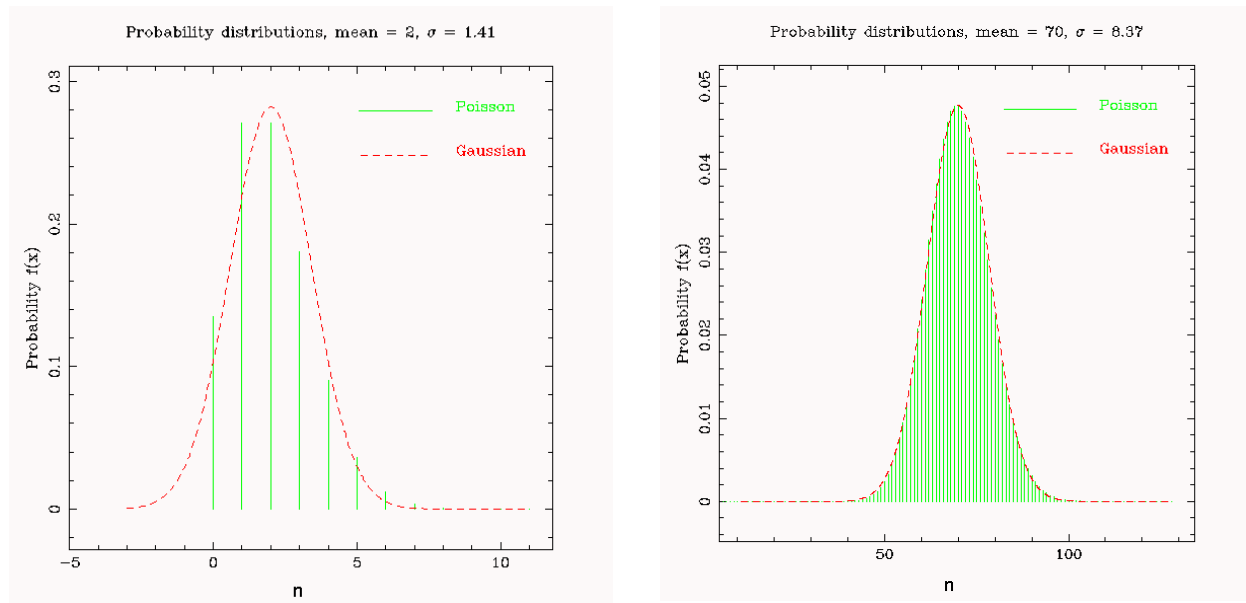


Fig. 4. Comparison of Poisson and Gaussian distributions for $\bar{n} = 2$ and 70. The approximation of using the Gaussian distribution improves as n increases.

Take $N \sim 50$ data points (observations of the value of n) and let N_b be the number of bins. Let the number of observations of n falling into the q -th bin be f_q . Then, as in the previous section,

$$\sum_{q=1}^{N_b} f_q = N \tag{10}$$

and the probability, P_q , is given by,

$$P_q = \frac{f_q}{N} \quad (11)$$

But this number does not correspond to the probability in equation (8), because the binning process in fact involves a summation over the probability distribution. The form of the Gaussian distribution that corresponds to (11) is,

$$P_q = \sum_{n_q - \Delta n/2}^{n_q + \Delta n/2} P(n) \approx P(n_q) \Delta n \quad (12)$$

where n_q is the value of n at the center of the q -th bin and $P(n)$ is given in (8).

The corresponding prediction for the frequency, f_q , is,

$$f_q = NP(n_q) \Delta n \quad (13)$$

Observing a Gaussian distribution

To do this part of the experiment you will use a radioactive ^{137}Cs source. Because of the unstable neutron to proton ratio, ^{137}Cs nuclei beta decay via the weak interaction to ^{137}Ba with the emission of an energetic electron (up to ~ 1 MeV) and an electron antineutrino – changing a neutron into a proton. This Cs decay has a 30 year half-life. The ^{137}Ba daughter nucleus is usually created in a metastable excited state $^{137\text{m}}\text{Ba}$, from which it decays in 3 minutes with the emission of a 662 KeV gamma ray (high energy photon). So your radioactive source will be emitting random ionizing energetic electrons and gamma rays at a roughly constant rate over the timescale of your experiment, because of the long half life of the ^{137}Cs source.

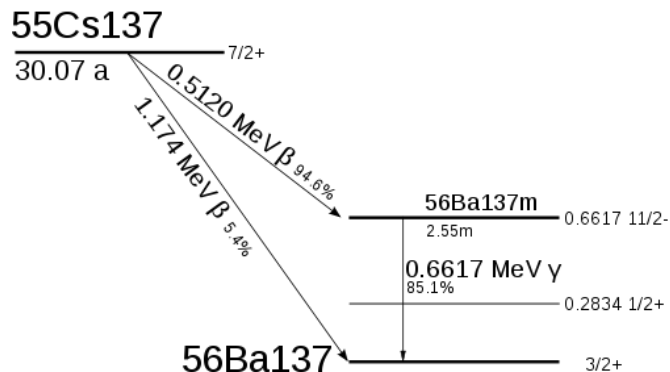


Fig. 5. Decay scheme of ^{137}Cs .

First, read the Radiation Safety notice at the end of this guide. Your TA will deliver your radioactive source – be sure to return it to your TA immediately after use. The Geiger tube is more sensitive to the ionizing electrons than the gamma-rays. Place the ^{137}Cs source beneath the Geiger tube in one of the slots. Note how the count rate changes as you vary the distance between the source and the GM tube. Consider all sources of error, including the accuracy of your timing. Based on this, choose a convenient time interval which will give approximately 1000 counts/interval for which you have accurate timing. Make $N=50$ different determinations of counts/chosen time interval. Then bin it into 6-7 bins. Describe your apparatus and place these data and the rest of your analysis in your lab book.

Find the mean and standard deviation of your binned data.

Since you used a histogram in the last section, this time plot f_q vs n_q as points on a standard linear plot, and show error bars of one standard deviation in the frequencies, f_q . (Recall that the standard deviation in the frequencies f_q is expected to be $\sqrt{f(q)}$. For example, if you had 100 counts in one of your frequency bins, the one standard deviation error bar would be ± 10 counts.) Then plot the prediction (13) on the same graph, treating n_q as a continuous variable, so that the prediction is shown as a smooth curve, while the measured frequencies are shown as points. What fraction of the data points have error bars that overlap the prediction? Is it what you expect? Discuss.

What is the mean of your count rate and the error on the mean?

Naturally all sources of radioactivity in your experiment produce uncorrelated counts in your detector, including the background radiation. The combination is nearly Gaussian distributed at high count rate. Suppose however you were interested in the statistics of your radioactive ^{137}Cs source itself in a situation where it was placed a long distance away from the detector such that the count rate from the source was only approximately five times the background rate. Describe how you would correct your data in this new experiment for the background and how the errors propagate.

V. Mean Life of the Muon

Unlike your ^{137}Cs source, many unstable nuclei and unstable fundamental particles decay quickly, and with the proper equipment you can measure the characteristic decay time. In this last exercise, by analyzing some muon decay data obtained by a student last year you will determine the mean lifetime of the muon, an unstable particle which occurs naturally in the environment due to cosmic ray showers. In analyzing these data, you will apply your knowledge of weighted least-squares fits and error propagation. But first, some basics of decay (see also Melissinos pp354-355) and of least-squares fitting.

For the nucleus of a radioactive isotope and for an unstable particle there is a constant probability per unit time that it will undergo decay. For example, in a sample of N atoms of a given isotope the number disintegrating in time dt is given by

$$dN = -\lambda N dt \quad (14)$$

where N is the total number of nuclei (or atoms) in the isotope sample and λ is the constant of proportionality called the decay constant. The solution to this equation is

$$N(t) = N_0 \exp(-\lambda t) \quad (15)$$

where N is the number present at $t = 0$. Evidently, the decay constant is related to the mean lifetime $\tau = 1/\lambda$ and the half-life $t_{1/2} = \ln 2/\lambda$. Both terminologies are used, so be careful which one is meant, and be sure to specify which of the two you quote in your lab book and write-up. Muon decay also follows such an exponential distribution with a mean lifetime of about 2 μsec (Melissinos p408).

On the web page for this experiment you will find a file called muondecay-2009student.xls (or something similar). This file contains a single vector of numbers each of which represents a single measurement of the time for a muon to decay (actually the time difference between the detection of a stopping muon and the emission of its decay electron or positron) in very fine bins of decay time going from 0-4096, where 4096 represents ten μsec . The file is for an entire 5-day run in which 13433 muon decays were measured (so the file is 13433 lines long, each line giving the measured decay time for that particular muon).

The first thing you should do is look at the distribution of decay times. To do this, make a histogram of the data, plotting number of events with a certain decay time vs the decay time in the fine bins (ADC channels) of the data acquisition system. A binned version of this is shown in Figure 6, but make your own plot and put it in your lab book. Clearly there are ranges of data you want to avoid, such as ADC channel numbers less than 125 and greater than 4075. Re-bin your accepted data into 200 bins of decay time.

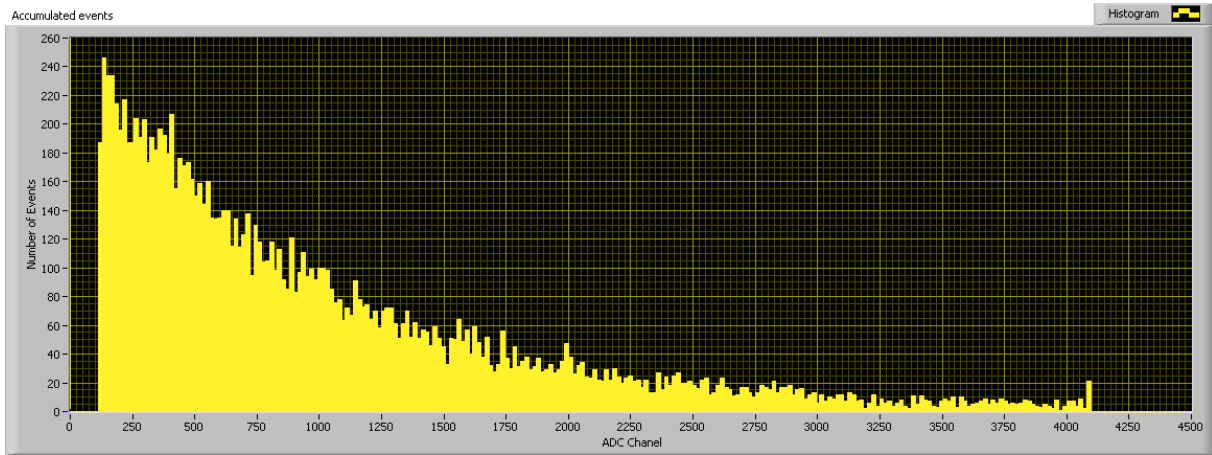


Fig. 6. Histogram of the distribution of all observed muon decay times for a 5 day run.

Since you are expecting an exponential decay, replot this binned accepted data as log number of events vs decay time bin. A pure exponential decay should appear as a straight line on the log-linear plot, and with some luck you could get a rough estimate of the mean decay time by fitting a straight line to this semi-log plot (try it). However, your semi-log plot probably shows a departure from this expected behavior, at the low count end. What is going on?

Suppose the muon experiment logic electronics is not perfect, and there is a constant probability of triggering on an unrelated radiation event instead of that stopped muon. Of course this background “accidentals” count rate is difficult to measure. It is possible to calculate this accidentals background from other data using the same apparatus. Suppose the student actually did this calculation and got a value of exactly 2 counts per time bin in your re-binned 200 bin data. If you account for this background, does your semi-log plot look better?

Now plot error bars on your 200 points. Assume purely Poisson statistics. You will find it convenient to form a second column of estimated Poisson error corresponding to each of the 200 time bins by taking the square root of the counts. Look at your plot now. Are the errors uncorrelated point-to-point? Do the points jump around by an amount consistent with the error bars? Obviously, some of the points have larger error bars than others and are thus less trustworthy.

Try to estimate the muon lifetime by taking into account these statistical errors. Make a weighted least-squares fit to your binned selected data, and solve for a best estimate mean muon lifetime. Suppose we want to fit a theoretically predicted function $y(t_i)$. If we have data points y_i each with its own standard deviation σ_i , then χ^2 is defined by

$$\chi^2 = \sum_{i=1}^N \frac{[y_i - y(t_i)]^2}{\sigma_i^2} \quad (16)$$

There are many algorithms for the minimization of χ^2 , as discussed in Bevington. Python has useful routines for exploring fits. Once a fit is achieved, how good is it? Your computer program should give you the value of χ^2 that results from the best fit λ . Since the mean of χ^2 is predicted to be the number of data points N (200 in this case), and its standard deviation to be $\sqrt{2N}$, you can compare and see if the value of χ^2 that you obtained is within these limits. Quote your answer for the “reduced χ^2 ” obtained by dividing χ^2 by the number of degrees of freedom (in this case $200-2=198$). A reduced χ^2 close to 1 is a good fit.

Quote your answer for the muon lifetime and its 95% confidence error. [There are corrections to this “raw” lifetime which depend on the nature of the detector and the ratio of positive to negative muons in the lab, but don’t worry about that for this exercise.] What if last year’s student got the wrong answer for the background! How might that affect your result? Try refitting for several values of the background to measure the sensitivity of your answer for the muon lifetime to this assumed background.

Suppose the student doing the muon experiment last year undertook the appropriate experiments and measured a background count and one standard deviation error (in your 200 bin system) of 2.0 ± 0.6 counts per bin. How would this propagate to your new quoted error? As you learned in the previous section, the error in your background counts n_b is the square root of this number. But this error cannot just be added to the error in your counts for each of your 200 bins. Instead, one must calculate the *propagation of errors*. i.e. for uncorrelated errors the errors add in quadrature. For example, in our case $y = n - n_b$, and so then $\sigma_y^2 = \sigma_n^2 + \sigma_b^2 = n + n_b$.

Finally, suppose you are not given the background accidental counts. You must then treat this as an unknown variable. You now have three parameters to fit: the amplitude of the exponential term, the decay constant in the exponential, and an additive constant (the unknown background). Make a weighted fit for this case and observe what happens to your resulting error in the muon mean lifetime. Also see what the best fit background level turns out to be! In your lab book and write-up quote also this revised estimate for the muon mean lifetime and its 95% confidence limits. In a real experiment you would also add another kind of error: your best estimate for systematic error. For example, suppose that the calibration of the 0-4096 ADC timescale was only accurate to 2%. Use this as an estimate of your systematic error and quote it separately.

Now that you have 10-15 pages of descriptions, data, and analysis in your lab notebook, write up a 5-8 page scientific project report on this entire counting statistics lab in the style recommended in the Reports section of the course website. Style files are supplied. You will use these data analysis techniques, and the scientific writing style, in your elective experiments and later in your career in science and engineering.

APPARATUS AND SAFETY NOTES FOR THE GEIGER COUNTER EXPERIMENT

1. Determination of Starting Potential, Threshold and Plateau

Connect the coaxial cable from the GM tube into the connector marked INPUT on the rear panel. Place HIGH VOLTAGE and MASTER switches in the OFF position and plug in power. Leave the HIGH VOLTAGE OFF and turn power ON. Check the counter (scalar) by placing the TEST/USE switch on TEST and the COUNT/STOP switch on COUNT. The scalar should now count line frequency pulses (60 counts/sec). Now that you are sure the scalar is working correctly, place a ^{137}Cs source in the second or third slot of the source holder. (Don't use the top slot as the counting rate is often too high). CAUTION: DO NOT TOUCH THE FACE OF THE GM TUBE DETECTOR AS THIS WILL BREAK THE THIN WINDOW AND RUIN THE TUBE!! Place the H.V. ADJUST in the MINIMUM position and turn on the high voltage. Now slowly increase the voltage until the scalar starts counting. The voltage where the scalar starts counting is called threshold of the tube. In doing the experiment, DO NOT EXCEED A VOLTAGE OF 1000 VOLTS [1200 VOLTS FOR SOME TUBES]. IF YOU HEAR A TICKING, TURN DOWN THE VOLTAGE!

2. Data Analysis

Some data analysis can be performed using the *KaleidaGraph* program. Far more capable data analysis programs are available in Python. *Igor* is also available to all students. If you prefer, other programs such as *MatLab*, *LabView*, and *IDL* may be used. Do not blindly use these programs; describe your analysis failures and successes in detail in your lab book and write-up, including fit equations and parameter errors.

3. Radiation Safety

In this lab you will work with a variety of radioactive sources. The textbook by Melissinos (page 485) has a section on radiation safety, which you should read before you use any of these sources. Most sources are relatively weak and involve little danger if you exercise minimal precautions. If you have any question about the amount of radiation near a source, there are Geiger counters in the lab, and you are encouraged to use them to measure radiation levels. The only significant danger from the sources in this lab arises if you ingest (eat or swallow) them. You will probably not intentionally eat any of the radioactive sources, but there is a small possibility that one of the seals may leak and this could ultimately contaminate something that you eat. Thus you should wash your hands after handling any radioactive source. Do not handle the sources with your fingers. It is safer to use the supplied tweezers. Eating or drinking of any food in the Roessler 154/156 lab area is strictly forbidden. Before using any radioactive sources you must check with an instructor or TA to be sure that you know how to handle them properly. Return your source to the TA before the end of the class period. Finally, after working with radioactive sources, it is recommended that you check your hands with a portable Geiger counter.