Class 1: DC Circuits

Topics:

- What this course treats: Art of Electronics
  DC circuits
  Today we will look at circuits made up entirely of
  - DC voltage sources (things whose output voltage is constant over time; things like a battery, or a lab power supply); and
  - resistors.

Sounds simple, and it is. We will try to point out quick ways to handle these familiar circuit elements. We will concentrate on one circuit fragment, the voltage divider.

Preliminary: What is “the art of electronics?”

Not an art, perhaps, but a craft. Here’s the Text’s formulation of what it claims to teach:

...the laws, rules of thumb, and tricks that constitute the art of electronics as we see it. (P. 1)

As you may have gathered, if you have looked at the text, this course differs from an engineering electronics course in concentrating on the “rules of thumb” and the “tricks.” You will learn to use rules of thumb and reliable tricks without apology. With their help you will be able to leave the calculator-bound novice engineer in the dust!

Two Laws

Text sec. 1.01

First, a glance at two laws: Ohm’s Law, and Kirchhoff’s Laws (V,I).

We rely on these rules continually, in electronics. Nevertheless, we rarely will mention Kirchhoff again. We use his observations implicitly. We will see and use Ohm’s Law a lot, in contrast (no one has gotten around to doing what’s demanded by the bumper sticker one sees around MIT: Repeal Ohm’s Law!)

Ohm’s Law: \( E = IR \)

\( E \) (old-fashioned term for voltage)

is analog of water pressure or ‘head’ of water

\( R \) describes the restriction of flow

\( I \) is the rate of flow (volume/unit time)

Figure N1.1: Hydraulic analogy: voltage as head of water, etc. Use it if it helps your intuition

The homely hydraulic analogy works pretty well, if you don’t push it too far—and if you’re not too proud to use such an aid to intuition.
Ohm’s is a very useful rule; but it applies only to things that behave like resistors. What are these? They are things that obey Ohm’s Law! (Sorry folks: that’s as deeply as we’ll look at this question, in this course.)

We begin almost at once to meet devices that do not obey Ohm’s Law (see Lab 1: a lamp; a diode). Ohm’s Law describes one possible relation between V and I in a component; but there are others.

As the text says, 

Crudely speaking, the name of the game is to make and use gadgets that have interesting and useful I vs V characteristics. (P. 4)

Kirchhoff’s Laws (V,I)

These two ‘laws’ probably only codify what you think you know through common sense:

Figure N1.2: Kirchhoff’s two laws

Applications of these laws: series and parallel circuits

Figure N1.3: Applications of Kirchhoff’s laws: Series and parallel circuits: a couple of truisms, probably familiar to you already

Query: Incidentally, where is the “loop” that Kirchhoff’s law refers to?

This is kind of boring. So, let’s hurry on to less abstract circuits: to applications—and tricks. First, some labor-saving tricks.
**Parallel Resistances:** calculating equivalent $R$

The *conductances* add:

$$\text{conductance}_{\text{total}} = \text{conductance}_1 + \text{conductance}_2 = \frac{1}{R_1} + \frac{1}{R_2}$$

![Parallel Resistors](image1.png)

Figure N1.4: Parallel resistors: the conductances add; unfortunately, the resistances don’t

This is the easy notion to remember, but not usually convenient to apply, for one rarely speaks of conductances. The notion “resistance” is so generally used that you will sometimes want to use the formula for the effective resistance of two parallel resistors:

$$R_{\text{tot}} = \frac{R_1 R_2}{R_1 + R_2}$$

Believe it or not, even this formula is messier than what we like to ask you to work with in this course. So we proceed immediately to some tricks that let you do most work in your head.

*Text sec. 1.02*

Consider some easy cases:

![Resistors](image2.png)

The first two cases are especially important, because they help one to *estimate* the effect of a circuit one can liken to either case. Labor-saving tricks that give you an estimate are not to be scorned: if you see an easy way to an estimate, you’re likely to make the estimate. If you have to work to hard to get the answer, you may find yourself simply *not* making the estimate.

In this course we usually are content with answers good to 10%. So, if two parallel resistors differ by a factor of ten, then we can ignore the larger of the two.

Let’s elevate this observation to a *rule of thumb* (our first). While we’re at it, we can state the equivalent rule for resistors in series.

**Parallel resistances: shortcuts**

![Parallel Resistors](image3.png)

In a parallel circuit, a resistor much *smaller* than others dominates. In a series circuit, the *large* resistor dominates.

Figure N1.6: Resistor calculation shortcut: parallel, series
Voltage Divider

Text sec. 1.03

At last we have reached a circuit that does something useful.

First, a note on labeling: we label the resistors “10k”; we omit “Ω.” It goes without saying. The “k” means kilo- or $10^3$, as you probably know.

One can calculate $V_{out}$ in several ways. We will try to push you toward the way that makes it easy to get an answer in your head.

Three ways:

1. Calculate the current through the series resistance; use that to calculate the voltage in the lower leg of the divider.

   ![Figure N1.8: Voltage divider: first method (too hard!): calculate current explicitly](image)

   $$I = \frac{V_{in}}{(R_1 + R_2)}$$
   Here, that’s $30\, \text{V} / 20\, \text{kΩ} = 1.5 \, \text{mA}$

   $$V_{out} = I \cdot R_2$$
   Here, that’s $1.5 \, \text{mA} \cdot 10\, \text{kΩ} = 15 \, \text{V}$

   That takes too long.

2. Rely on the fact that $I$ is constant in top and bottom, but do that implicitly. If you want an algebraic argument, you might say,

   $$\frac{V_2}{(V_1 + V_2)} = \frac{IR_2}{I[R_1 + R_2]} = \frac{R_2}{(R_1 + R_2)}$$

   or,

   $$(1), \quad V_{out} = V_{in} \left\{ \frac{R_2}{(R_1 + R_2)} \right\}$$

   In this case, that means

   $$V_{out} = V_{in} \left( \frac{10k}{20k} \right) = \frac{V_{in}}{2}$$

   That’s much better, and you will use formula $(1)$ fairly often. But we would like to push you not to memorize that equation, but instead to—
3. Say to yourself in words how the divider works: something like,

Since the currents in top and bottom are equal, the voltage drops are proportional to the resistances (later, impedances—a more general notion that covers devices other than resistors).

So, in this case, where the lower $R$ makes up half the total resistance, it also will show half the total voltage.

For another example, if the lower leg is 10 times the upper leg, it will show about 90% of the input voltage (10/11, if you're fussy, but 90%, to our usual tolerances).

**Loading, and “output impedance”**

*Text sec. 1.05,*

Now—after you’ve calculated $V_{out}$ for the divider—suppose someone comes along and puts in a third resistor:

*Text exercise 1.9*

![Figure N1.10: Voltage divider loaded](image)

(Query: Are you entitled to be outraged? Is this no fair?) Again there is more than one way to make the new calculation—but one way is tidier than the other.

**Two possible methods:**

1. **Tedious Method:**

   *Text exercise 1.19*

   Model the two lower R’s as one R; calculate $V_{out}$ for this new voltage divider:

   ![Figure N1.11: Voltage divider loaded: load and lower R combined in model](image)

   The new divider delivers $1/3 \ V_{in}$

   That’s reasonable, but it requires you to draw a new model to describe each possible loading.
2. Better method: Thevenin’s.

Thevenin Model

Thevenin’s good idea:
Model the actual circuit (unloaded) with a simpler circuit—the Thevenin model—which is an idealized voltage source in series with a resistor. One can then see pretty readily how that simpler circuit will behave under various loads.

![Figure N1.12: Thevenin Model: perfect voltage source in series with output resistance](image)

Here’s how to calculate the two elements of the Thevenin model:

\[ V_{\text{Thevenin}}: \]
Just \( V_{\text{open circuit}} \): the voltage out when nothing is attached (“no load”)

\[ R_{\text{Thevenin}}: \]
\( \text{Defined as the quotient of } V_{\text{Thevenin}} / I_{\text{short-circuit}}, \text{ which is the current that flows from the circuit output to ground if you simply short the output to ground.} \)

In practice, you are not likely to discover \( R_{\text{Thev}} \) by so brutal an experiment; and if you have a diagram of the circuit to look at, there is a much faster shortcut:

**Shortcut calculation of** \( R_{\text{Thev}} \)

Given a circuit diagram, the fastest way to calculate \( R_{\text{Thev}} \) is to see it as the parallel resistance of the several resistances viewed from the output.

![Figure N1.13: \( R_{\text{Thev}} = R_1 \) parallel \( R_2 \)](image)

(This formulation assumes that the voltage sources are ideal, incidentally; when they are not, we need to include their output resistance. For the moment, let’s ignore this complication.)
You saw this result above, but this still may strike you as a little odd: why should \( R_1 \), going up to the positive supply, be treated as parallel to \( R_2 \)? Well, suppose the positive supply were set at 0 volts. Then surely the two resistances would be in parallel, right?

Or suppose a different divider (chosen to make the numbers easy): twenty volts divided by two 10k resistors. To discover the impedance at the output, do the usual experiment (one that we will speak of again and again):

A general definition and procedure to determine impedance at a point:

To discover the impedance at a point:
apply a \( \Delta V \); find \( \Delta I \).

The quotient is the impedance.

This you will recognize as just a "small-signal" or "dynamic" version of Ohm's Law.

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In this case 1 mA was flowing before the wiggle. After we force the output up by 1v, the currents in top and bottom resistors no longer match: upstairs: 0.9 mA; downstairs, 1.1 mA. The difference must come from you, the wiggler.

Result: impedance = \( \Delta V / \Delta I = 1 \text{v} / 0.2 \text{mA} = 5 \text{k} \)

And—happily—that is the parallel resistance of the two R's. Does that argument make the result easier to accept?

You may be wondering why this model is useful. Here is one way to put the answer, though probably you will remain skeptical until you have seen the model at work in several examples: Any non-ideal voltage source "droops" when loaded. How much it droops depends on its "output impedance". The Thevenin equivalent model, with its \( R_{\text{Thevenin}} \), describes this property neatly in a single number.

**Applying the Thevenin model**

First, let's make sure Thevenin had it right: let's make sure his model behaves the way the original circuit does. We found that the 10k, 10k divider from 30 volts, which put out 15v when not loaded, drooped to 10V under a 10k load. Does the model do the same?
Yes, the model droops to the extent the original did: down to 10 V. What the model provides that the original circuit lacked is that single value, $R_{\text{Thev}}$, expressing how droopy/stiff the output is.

If someone changed the value of the load, the Thevenin model would help you to see what droop to expect; if, instead, you didn’t use the model and had to put the two lower resistors in parallel again and recalculate their parallel resistance, you’d take longer to get each answer, and still you might not get a feel for the circuit’s output impedance.

Let’s try using the model on a set of voltage sources that differ only in $R_{\text{Thev}}$. At the same time we can see the effect of an instrument’s input impedance.

Suppose we have a set of voltage dividers, dividing a 20 V input by two. Let’s assume that we use 1% resistors (value good to ±1%).

\[
\begin{array}{c}
\text{Figure N1.16: A set of similar voltage dividers: same } V_{\text{Th}}, \text{ differing } R_{\text{Th}}'s \\

V_{\text{Thev}} \text{ is obvious, and is the same in all cases. } R_{\text{Th}} \text{ evidently varies from divider to divider.}
\end{array}
\]

Suppose now that we try to measure $V_{\text{out}}$ at the output of each divider. If we measured with a perfect voltmeter, the answer in all cases would be 10 V. (Query: is it 10.000 V? 10.0 V?)

But if we actually perform the measurement, we will encounter the $R_{\text{in}}$ of our imperfect lab voltmeters. Let’s try it with a VOM (“volt-ohm-meter,” the conventional name for the old-fashioned “analog” meter, which gives its answers by deflecting its needle to a degree that forms an analog to the quantity measured), and then with a DVM (“digital voltmeter,” a more recent invention, which usually can measure current and resistance as well as voltage, despite its name; both types sometimes are called simply “multimeters”).

Suppose you poke the several divider outputs, beginning from the right side, where the resistors are 1 kΩ. Here’s a table showing what we find, at three of the dividers:

<table>
<thead>
<tr>
<th>$R$ values, divider</th>
<th>Measured $V_{\text{out}}$</th>
<th>Inference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 k</td>
<td>9.95</td>
<td>within R tolerance</td>
</tr>
<tr>
<td>10 k</td>
<td>9.76</td>
<td>loading barely apparent</td>
</tr>
<tr>
<td>100 k</td>
<td>8.05</td>
<td>loading obvious</td>
</tr>
</tbody>
</table>

The 8.05 V reading shows such obvious loading—and such a nice round number, if we treat it as “8 V”—that we can use this to calculate the meter’s $R_{\text{in}}$ without much effort:

\[
\begin{array}{c}
\text{Figure N1.17: VOM reading departs from ideal; we can infer } R_{\text{in}} \text{ from this reading.}
\end{array}
\]
As usual, one has a choice now, whether to pull out a formula and calculator, or whether to try, instead, to do the calculation "back-of-the-envelope" style. Let's try the latter method.

First, we know that $R_{Thev}$ is 100k parallel 100k: 50k. Now let's talk our way to a solution (an approximate solution: we'll treat the measured $V_{out}$ as just "8 volts":

The meter shows us 8 parts in 10; across the divider's $R_{Thev}$ (or call it "$R_{out}$" we must be dropping the other 2 parts in 10. The relative sizes of the two resistances are in proportion to these two voltage drops: 8 to 2, so $R_{in-VOM}$ must be $4 \cdot R_{Thev}$: 200k.

If we squint at the front of the VOM, we'll find a little notation, 20,000 ohms/volt
That specification means that if we used the meter on its 1 V scale (that is, if we set things so that an input of 1 volt would deflect the needle fully), then the meter would show an input resistance of 20k. In fact, it's showing us 200k. Does that make sense? It will when you've figured out what must be inside a VOM to allow it to change voltage ranges: a set of big series resistors. You'll understand this fully when you have done problem 1.8 in the text; for now, take our word for it: our answer, 200k, is correct when we have the meter set to the 10 V scale, as we do for this measurement.

This is probably a good time to take a quick look at what's inside a multimeter—VOM or DVM:
How a meter works:
- Depends on type.
- depends whether the basic works of the meter sense current or voltage.

- analog: senses current

- digital: senses voltage

The VOM specification, 20,000 ohms/volt, describes the sensitivity of the meter movement—the guts of the instrument. This movement puts a fairly low ceiling on the VOM's input resistance at a given range setting.
Let's try the same experiment with a DVM, and let's suppose we get the following readings:

<table>
<thead>
<tr>
<th>R values, divide</th>
<th>Measured V\textsubscript{out}</th>
<th>Inference</th>
</tr>
</thead>
<tbody>
<tr>
<td>100k</td>
<td>9.92</td>
<td>within R tolerance</td>
</tr>
<tr>
<td>1M</td>
<td>9.55</td>
<td>loading apparent</td>
</tr>
<tr>
<td>10M</td>
<td>6.69</td>
<td>loading obvious</td>
</tr>
</tbody>
</table>

![Figure N1.20: DVM reading departs from ideal; we can infer $R_{\text{in-DVM}}$](image)

Again let's use the case where the droop is obvious; again let's talk our way to an answer:

This time $R_{\text{Th}}$ is 5M; we’re dropping 2/3 of the voltage across $R_{\text{in-DVM}}$, 1/3 across $R_{\text{Th}}$. So, $R_{\text{in-DVM}}$ must be $2 \times R_{\text{Th}}$, or 10M.

If we check the data sheet for this particular DVM we find that its $R_{\text{in}}$ is specified to be “≥ 10M, all ranges.” Again our readings make sense.

*VOM vs DVM: a conclusion?*

Evidently, the DVM is a better voltmeter, at least in its $R_{\text{in}}$—as well as much easier to use. As a current meter, however, it is no better than the VOM: it drops 1/4 v full scale, as the VOM does; it measures current simply by letting it flow through a small resistor; the meter then measures the voltage across that resistor.

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**Digression on ground**

The concept “ground” (“earth,” in Britain) sounds solid enough. It turns out to be ambiguous. Try your understanding of the term by looking at some cases:

![Figure N1.21: Ground in two senses](image)

Query: what is the resistance between points A and B? (Easy, if you don’t think about it too hard.) We know that the ground symbol means, in any event, that the bottom ends of the two resistors are electrically joined. Does it matter whether that point is also tied to the pretty planet we live on? It turns out that it does not.
And where is "ground" in this circuit:

**Figure N1.22: Ground in two senses, revisited**

*Local ground* is what we care about: the common point in our circuit that we arbitrarily choose to call zero volts. Only rarely do we care whether or not that local reference is tied to a spike driven into the earth. But, *be warned*, sometimes you are confronted with lines that are tied to world ground—for example, the ground clip on a scope probe, and the "ground" of the breadboards that we use in the lab; then you must take care not to clip the scope ground to, say, +15 on the breadboard.

**Generalizing what we've learned of $R_{in}$ and $R_{out}$**

The voltage dividers whose outputs we tried to measure introduced us to a problem we will see over and over again: some circuit tries to "drive" a load. To some extent, the load changes the output. We need to be able to predict and control this change. To do that, we need to understand, first, the characteristic we call $R_{in}$ (this rarely troubles anyone) and, second, the one we have called $R_{Thevenin}$ (this one takes longer to get used to). Next time, when we meet frequency-dependent circuits, we will generalize both characteristics to "$Z_{in}$" and "$Z_{out}$."

Here we will work our way to another rule of thumb; one that will make your life as designer relatively easy. We start with a goal: *Design goal:* When circuit A drives circuit B: arrange things so that B loads A lightly enough to cause only insignificant attenuation of the signal. And this goal leads to the rule of thumb:

**Design rule of thumb:**

*When circuit A drives circuit B:*

Let $R_{out}$ for A be $\leq 1/10 R_{in}$ for B

**Figure N1.23: Circuit A drives circuit B**

How does this rule get us the desired result? Look at the problem as a familiar voltage divider question. If $R_{outA}$ is much smaller than $R_{inB}$, then the divider delivers nearly all of the original signal. If the relation is $1 : 10$, then the divider delivers $10/11$ of the signal: attenuation is just under $10\%$, and that's good enough for our purposes.

We like this arrangement not just because we like big signals. (If that were the only concern, we could always boost the output signal, simply amplifying it.) We like this
arrangement above all because it allows us to design circuit-fragments independently: we can design A, then design B, and so on. We need not consider A, B as a large single circuit. That’s good: makes our work of design and analysis lots easier than it would be if we had to treat every large circuit as a unit.

An example, with numbers: What $R_{\text{Thev}}$ for droop of < 10%? What R’s, therefore?

![Figure N1.24: One divider driving another: a chance to apply our rule of thumb](image)

The effects of this rule of thumb become more interesting if you extend this chain: from A and B, on to C.

![Figure N1.25: Extending the divider: testing the claim that our rule of thumb lets us consider one circuit fragment at a time](image)

As we design C, what $R_{\text{Thev}}$ should we use for B? Is it just 10K parallel 10K? That’s the answer if we can consider B by itself, using the usual simplifying assumptions: source ideal ($R_{\text{out}} = 0$) and load ideal ($R_{\text{in}}$ infinitely large).

But should we be more precise? Should we admit that the upper branch really looks like 10K + 2K: 12k? Oh dear! That’s 20% different. Is our whole scheme crumbling? Are we going to have to look all the way back to the start of the chain, in order to design the next link? Must we, in other words, consider the whole circuit at once, not just the fragment B, as we had hoped?

No. Relax. That 20% error gets diluted to half its value: $R_{\text{Thev}}$ for B is 10k parallel 12k, but that’s a shade under 5.5k. So—fumes of relief!—we need not look beyond B. We can, indeed, consider the circuit in the way we had hoped: fragment by fragment.

If this argument has not exhausted you, you might give our claim a further test by looking in the other direction: does C alter B’s input resistance appreciably (>10%)? You know the answer, but confirming it would let you check your understanding of our rule of thumb and its effects.
Chapter 1: Worked Examples: Resistors & Instruments

Five worked examples:

1. Design a voltmeter and ammeter from bare meter movement
2. Effects of instrument imperfections, in first lab (L1-1)
3. Thevenin models
4. $R_{in}, R_{out}$
5. Effect of loading

1. Design a Voltmeter, Current Meter

Text sec. 1.04, ex. 1.8, p. 10

Problem: Modify a meter movement to form a voltmeter and ammeter

A 50μA meter movement has an internal resistance of 5k. What shunt resistance is needed to convert it to a 0-1 amp meter? What series resistance will convert it to a 0-10 volt meter?

This exercise gives you a useful insight into the instrument, of course, but it also will give you some practice in judging when to use approximations: how precise to make your calculations, to say this another way.

1 amp meter

"50μA meter movement" means that the needle deflects fully when 50μA flows through the movement (a coil that deflects in the magnetic field of a permanent magnet: see Class 1 notes for a sketch). The remaining current must bypass the movement; but the current through the movement must remain proportional to the whole current.

Such a long sentence makes the design sound complicated. In fact, as probably you have seen all along, the design is extremely simple: just add a resistance in parallel with the movement (this is the "shunt" mentioned in the problem):

What value?

Well, what else do we know? We know the resistance of the meter movement. That characteristic plus the full-scale current tell us the full-scale voltage drop across the movement: that's

$$V_{movement(full-scale)} = I_{full-scale} \cdot R_{movement} = 50 \mu A \cdot 5k \Omega = 250 \text{ mV}$$

Now we can choose $R_{shunt}$, since we know current and voltage that we want to see across the parallel combination. At this point we have a chance to work too hard, or instead to use a
sensible approximation. The occasion comes up as we try to answer the question, ‘How much current should pass through the shunt?’

One possible answer is ‘1A less 50µA, or 0.99995 A.’

Another possible answer is ‘1A.’

Which do you like? If you’re fresh from a set of Physics courses, you may be inclined toward the first answer. If we take that, then the resistance we need is

\[ R = \frac{V_{\text{full-scale}}}{I_{\text{full-scale}}} = \frac{250 \text{ mV}}{0.99995 \text{ A}} = 0.2500125 \text{ Ω} \]

Now in some settings that would be a good answer. In this setting, it is not. It is a very silly answer. That resistor specification claims to be good to a few parts in a million. If that were possible at all, it would be a preposterous goal in an instrument that makes a needle move so we can squint at it.

So we should have chosen the second branch at the outset: seeing that the 50µA movement current is small relative to the the 1A total current, we should then ask ourselves, ‘About how small (in fractional or percentage terms)?’ The answer would be ‘50 parts in a million.’ And that fraction is so small relative to reasonable resistor tolerances that we should conclude at once that we should neglect the 50µA.

Neglecting the movement current, we find the shunt resistance is just 250mV/1A = 250 mΩ. In short, the problem is very easy if we have the good sense to let it be easy. You will find this happening repeatedly in this course: if you find yourself churning through a lot of math, and especially if you are carrying lots of digits with you, you’re probably overlooking an easy way to do the job. There is no sense carrying all those digits and then having to reach for a 5% resistor and 10% capacitor.

Voltmeter

Here we want to arrange things so that 10V applied to the circuit causes a full-scale deflection of the movement. Which way should we think of the cause of that deflection—‘50µA flowing,’ or ‘250 mV across the movement?’

Either is fine. Thinking in voltage terms probably helps one to see that most of the 10V must be dropped across some element we are to add, since only 0.25V will be dropped across the meter movement. That should help us sketch the solution:

![Figure X1.2: Voltmeter: series resistance needed](image)

What series resistance should we add? There are two equivalent ways to answer:

1. The resistance must drop 9.75 volts out of 10, when 50µA flows; so \( R = \frac{9.75 \text{ V}}{50 \mu \text{A}} = 195k \text{ Ω} \)

2. Total resistance must be \( \frac{10 \text{ V}}{50 \mu \text{A}} = 200k \text{ Ω} \). The meter movement looks like 5k, we were told; so we need to add the difference, 195kΩ.

If you got stung on the first part of this problem, giving an answer like “0.2500125Ω,” then you might be inclined to say, ‘Oh, 50µA is very small; the meter is delicate, so I’ll neglect it. I’ll put in a 200k series resistor, and be just a little off.’

Well, just to keep you off-balance, we must now push you the other way: this time, “50µA,” though a small current is not negligibly small, because it is not to be compared with some much larger current. On the contrary, it is the crucial characteristic we need to work with: it determines the value of the series resistor. And we should not say ‘200k is
close enough,’ though 195k is the exact answer. The difference is 2.5%: much less than what we ordinarily worry about in this course (because we need to get used to components with 5 and 10% tolerances); but in a meter it’s surely worth a few pennies extra to get a 1% resistor: a 195k.

2. Lab 1-1 questions: working around imperfections of instruments

The very first lab exercise asks you to go through the chore of confirming Ohm’s Law. But it also confronts you at once with the difficulty that you cannot quite do what the experiment asks you to do: measure $I$ and $V$ in the resistor simultaneously. Two placements of the DVM are suggested (one is drawn, the other hinted at):

![Lab 1-1 setup: DVM and VOM cannot both measure the relevant quantity](image)

**Figure X1.3: Lab 1-1 setup: DVM and VOM cannot both measure the relevant quantity**

*A Qualitative View*

Just a few minutes’ reflection will tell you that the voltage reading is off, in the circuit as drawn; moving the DVM solves that problem (above), but now makes the current reading inaccurate.

*A Quantitative View*

Here’s the problem we want to spend a few minutes on:

**Problem:**

*Errors caused by the Meters*

If the analog meter movement is as described in the Text’s problem 1.8, what percentage error in the voltage reading results, if the voltage probe is connected as shown in the figure for the first lab 1 experiment, when the measured resistor has the following values. Assume that you are applying 20 volts, and that you can find a meter setting that lets you get full-scale deflection in the current meter.

- $R = 20k$ ohms.
- $R = 200$ ohms.
- $R = 2M$ ohms.

Same question, but concerning current measurement error, if the voltmeter probe is moved to connect directly to the top of the resistor, for the same resistor values. Assume the DVM has an input resistance of 20 M ohms.
Errors in Voltage readings

The first question is easier than it may appear. The error we get results from the voltage drop across the current meter; but we know what that drop is, from problem 1.8: full-scale: 0.25V. So, the resistor values do not matter. Our voltage readings always are high by a quarter volt, if we can set the current meter to give full-scale deflection. The value of the resistor being measured does not matter.

When the DVM reads 20V, the true voltage (at the top of the resistor) is 19.75V. Our voltage reading is high by 0.25V/19.75V—about 0.25/20 or 1 part in 80: 1.25% (If we applied a lower voltage, the voltage error would be more important, assuming we still managed to get full-scale deflection from the current meter, as we might be able to by changing ranges).

Errors in Current readings

If we move the DVM to the top of the resistor, then the voltage reading becomes correct: we are measuring what we meant to measure. But now the current meter is reading a little high: it measures not only the resistor current but also the DVM current, which flows parallel to the current in $R$.

The size of this error depends directly on the size of $R$ we are measuring. You don’t even need a pencil and paper to calculate how large the errors are:

\[
\text{~It=}\frac{UI_{o}}{R}\text{~}
\]

Figure X1.4: DVM causes current-reading error: how large? % error same as ratio of $R$ to $R_{DVM}$

If $R$ is 20kΩ —and the DVM looks like 20M—then one part in a thousand of the total current flows through the DVM: the current reading will be high by 0.1%.

If $R$ is 200Ω then the current error is minute: 1 part in 100,000: 0.001%.

If $R$ is 2MΩ then the error is large: 1 part in ten.

Conclusion?

There is no general answer to the question, ‘Which is the better way to place the DVM in this circuit?’ The answer depends on $R$, on the applied voltage and on the consequent ammeter range setting.

And before we leave this question, let’s notice the implication of that last phrase: the error depends on the VOM range setting. Why? Well, this is our first encounter with the concept we like to call Electronic Justice, or the principle that The Greedy Will Be Punished. No doubt these names mystify you, so we’ll be specific: the thought is that if you want good resolution from the VOM, you will pay a price: the meter will alter results more than if you looked for less resolution:
If you want the current meter needle to swing nearly all the way across the dial (giving best resolution: small changes in current cause relatively large needle movement), then you’ll get nearly the full-scale 1/4-volt drop across the ammeter. The same goes for the DVM as ammeter, if you understand that ‘full scale’ for the DVM means filling its digital range: “3 1/2 digits,” as the jargon goes: the “half digit” is a character that takes only the values zero or one. So, if you set the DVM current range so that your reading looks like

you have poor resolution: about one percent. If you are able to choose a setting that makes the same current look like 0.930, you’ve improved resolution 10X. But you have also increased the voltage drop across the meter by the same factor; for the DVM, like the analog VOM drops 1/4V full-scale, and proportionately less for smaller “deflection” (in the VOM) or smaller fractions of the full-scale range (for the DVM).

3. Thevenin Models

**Problem: Thevenin Models**

Draw Thevenin Models for the following circuits. Give answers to 10% and to 1%

![Thevenin Models Diagram](attachment:image.png)

Some of these examples show typical difficulties that can slow you down until you have done a lot of Thevenin models.

The leftmost circuit is most easily done by temporarily redefining *ground*. That trick puts the circuit into entirely familiar form:
The only difficulty that the middle circuit presents comes when we try to approximate. The 1% answer is easy, here. The 10% answer is tricky. If you have been paying attention to our exhortations to use 10% approximations, then you may be tempted to model each of the resistor blocks with the dominant $R$: the small one, in the parallel case, the big one in the series case:

Unfortunately, this is a rare case when the errors gang up on us; we are obliged to carry greater precision for the two elements that make up the divider.

This example is not meant to make you give up approximations. It makes the point that it’s the result that should be good to the stated precision, not just the intermediate steps.

The current source shown here probably looks queer to you. But you needn’t understand how to make one to see its effect; just take it on faith that it does what’s claimed: sources (squirts) a fixed current, down toward the negative supply. The rest follows from Ohm’s Law. (In Chapter 2 you will learn to design these things—and you will discover that some devices just do behave like current sources without being coaxed into it: transistors behave this way—both bipolar and FET.)
The point that the current source shows a very high output impedance helps to remind us of the definition of impedance: always the same: $\Delta V/\Delta I$. It is better to carry that general notion with you than to memorize a truth like 'Current sources show high output impedance.' Recalling that definition of impedance, you can always figure out the current source’s approximate output impedance (large versus small); soon you will know the particular result for a current source, just because you will have seen this case repeatedly.

4. ‘Looking through’ a circuit fragment, and $R_{in}$, $R_{out}$

**Problem:** $R_{in}$, $R_{out}$

What are $R_{in}$, $R_{out}$ at the indicated points?

![Diagram](image.png)

**Figure X1.10:** Determining $R_{in}$, $R_{out}$; you need to decide what’s beyond the circuit to which you’re connecting

It’s clear what $R_{in}$ the divider should show: just $R_1 + R_2$: $10k + 10k = 20k$. But when we say that are we answering the right question? Isn’t the divider surely going to drive something down the line? If not, why was it constructed?

The answer is Yes, it is going to drive something else—the load. But that something else should show its own $R_{in}$ high enough so that it does not appreciably alter the result you got when you ignored the load. If we follow our $10X$ rule of thumb (see the end of Class 1 notes), you won’t be far off this idealization: less than $10\%$ off. To put this concisely, you might say simply that we assume an ideal load: a load with infinite input impedance.

![Diagram](image.png)

**Figure X1.11:** $R_{in}$; we need an assumption about the load that the circuit drives, if we are to determine $R_{in}$
Here the same problem arises—and we settle it in the same way: by assuming an ideal source. The difficulty, again, is that we need to make some assumption about what is on the far side of the divider if we are to determine $R_{\text{out}}$:

![Figure X1.12: $R_{\text{out}}$: we need an assumption about the source that drives the circuit, if we are to determine $R_{\text{out}}$](image)

### 4. Effects of loading

#### Problem: Effects of loading

What is the voltage at $X$—

![Figure X1.13: $V_{\text{out}}$: calculated versus measured](image)

with no load attached?

When measured with a VOM labeled “10,000 ohms/volt?”

When measured with a scope whose input resistance is 1 MΩ?

This example recapitulates a point made several times over in the the first day’s class notes, as you recognize. **Reminder:** The “...ohms/volt” rating implies that on the 1-volt scale (that is, when 1V causes full deflection of the meter) the meter will present that input resistance. What resistance would the meter present when set to the 10 volt scale?

We start, as usual, by trying to reduce the circuit to familiar form. The Thevenin model does that for us. Then we add meter or scope as load, forming a voltage divider, and see what voltage results:
You will go through this general process again and again, in this course: reduce an unfamiliar circuit diagram to one that looks familiar. Sometimes you will do that by merely redrawing or rearranging the circuit; more often you will invoke a model, and often that model will be Thevenin’s.
1-1. Ohm's Law

Figure L1.1: Circuit for measurement of resistor's I vs V

First, the pedestrian part of this exercise: Verify that the resistor obeys Ohm's law, by measuring V and I for a few voltages.

A preliminary note on procedure

The principal challenge here is simply to get used to the breadboard and the way to connect instruments to it. We do not expect you to find Ohm's Law surprising. Try to build your circuit on the breadboard, not in the air. Novices often begin by suspending a resistor between the jaws of alligator clips that run to power supply and meters. Try to do better: plug the resistor into the plastic breadboard strip. Bring power supply and meters to the breadboard through jacks (banana jacks, if your breadboard has them); then plug a wire into the breadboard so as to join resistor, for example, to banana jack. Below is a sketch of the poor way and better way to build a circuit.
Have your instructor demonstrate which holes are connected to which, how to connect voltages and signals from the outside world, etc.

This is also the right time to begin to establish some conventions that will help you keep your circuits intelligible:

- Try to build your circuit so that it looks like its circuit diagram:
  
  - Let signal flow in from left, exit on right (in this case, the “signal” is just \( V \); the “output” is just \( I \), read on the ammeter);
  
  - Place ground on a horizontal breadboard bus strip below your circuit; place the positive supply on a similar bus above your circuit. When you reach circuits that include negative supply, place that on a bus strip below the ground bus.
  
  - Use color coding to help you follow your own wiring: use black for ground, red for the positive supply. Such color coding helps a little now, a lot later, when you begin to lay out more complicated digital circuits.

Use a variable regulated dc supply, and the hookup shown in the first figure, above, Fig. L1.1. Note that voltages are measured between points in the circuit, while currents are
measured through a part of a circuit. Therefore you usually have to break the circuit to measure a current.

Measure a few values of $V$ and $I$ for the 20k resistor (note: you may well find no 20k resistor in your kit. Don’t panic. Consider how to take advantage of some 10k’s.) Next try a 10k resistor instead, and sketch the two curves that these resistors define on a plot of $I$ vs $V$. You may be disinclined to draw these “curves,” because you knew without doing this experiment what they would look like. Fair enough. But we encourage you to draw the plot for contrast with the devices you will meet next—which interest us just because their curves do not look like those of a resistor: just because these other devices do not obey Ohm’s Law.

**Effects of the instruments on your readings**

Now that you have done what we called the pedestrian part of the experiment, consider a couple of practical questions that arise in even this simplest of “experiments.”

**A Qualitative View**

The voltmeter is not measuring the voltage at the place you want, namely across the resistor. Does that matter? How can you fix the circuit so the voltmeter measures what you want? When you’ve done that, what about the accuracy of the current measurement? Can you summarize by saying what an ideal voltmeter (or ammeter) should do to the circuit under test? What does that say about its “internal resistance”?

**A Quantitative View**

How large is each error, given a 20k resistor. Which of the two alternative hookups is preferable, therefore? Would you have reached the same conclusion if the resistor had been 20MΩ?

(You will find this question pursued in one of the Worked Examples.)

**Two Nonlinear Devices: (Ohm’s Law Defied!)**

1-2. An incandescent lamp

Now perform the same measurement ($I$ vs $V$) for a #47 lamp. Use the 100mA and 500mA scales on your VOM. Do not exceed 6.5 volts! Again you need only a few readings. Again we suggest you plot your results on the drawing you used to show the resistor’s behavior. Get enough points to show how the lamp diverges from resistor-like performance.

What is the “resistance” of the lamp? Is this a reasonable question? If the lamp’s filament is made of a material fundamentally like the material used in the resistors you tested earlier, what accounts for the funny shape of the lamp’s $V$ vs $I$ curve?
1-3. The Diode

Here is another device that does not obey Ohm's law: the **diode**. (We don't expect you to understand how the diode works yet; we just want you to meet it, to get some perspective on Ohm’s Law devices: to see that they constitute an important but special case).

We need to modify the test setup here, because you can’t just stick a voltage across a diode, as you did for the resistor and lamp above. You’ll see why after you’ve measured the diode’s V vs I. Do that by wiring up the circuit shown below.

![Diode VI measuring circuit](image)

In this circuit you are applying a current, and noting the diode voltage that results; earlier, you applied a voltage and read resulting current. The 1k resistor limits the current to safe values. Vary R (use a 100k variable resistor (usually called a *potentiometer* or "pot" even when wired, as here, as a variable resistor), a resistor substitution box, or a selection of various fixed resistors), and look at I vs V, sketching the plot in two forms: linear and "semi-log."

First, get an impression of the shape of the linear plot; just four or five points should define the shape of the curve. Then draw the same points on a semi-log plot, which compresses one of the axes. (Evidently, it is the fast-growing current axis that needs compressing, in this case.) If you have some semi-log paper use it. If you don’t have such paper, you can use the small version laid out below. The point is to see the pattern. You will see this shape again in Lab 5 when you let the scope plot diode and transistor characteristics for you.

---

1. Well, you can; but you can’t do it twice with one diode!
See what happens if you reverse the direction of the diode.

How would you summarize the $V$ vs $I$ behavior of a diode?

Now explain what would happen if you were to put 5 volts across the diode (Don’t try it!). Look at a diode data sheet, if you’re curious: see what the manufacturer thinks would happen. The data sheet won’t say “Boom” or “Pffft,” but that is what it will mean.

We’ll do lots more with this important device; see, e.g., secs. 1.25-1.31 in the text, and Lab 3.

1-4. Voltage Divider

Construct the voltage divider shown above (this is the circuit described in Exercise 1.9 (p. 10 of the text)). Apply $V_{in} = 15$ volts (use the dc voltages on the breadboard). Measure the (open circuit) output voltage. Then attach a 10k load and see what happens.

Now measure the short circuit current. (That means “short the output to ground, but make the current flow through your current meter. Don’t let the scary word “short” throw you: the current in this case will be very modest. You may have grown up thinking “a short blows a fuse.” That’s a good generalization around the house, but it often does not hold in electronics.)

From $I_{\text{Short Circuit}}$ and $V_{\text{Open Circuit}}$ you can calculate the Thevenin equivalent circuit.

Now build the Thevenin equivalent circuit, using the variable regulated dc supply as the voltage source, and check that its open circuit voltage and short circuit current match those
of the circuit that it models. Then attach a 10k load, just as you did with the original voltage divider, to see if it behaves identically.

A Note on Practical use of Thevenin Models

You will rarely do again what you just did: short the output of a circuit to ground in order to discover its $R_{\text{Thevenin}}$ (or "output impedance," as we soon will begin to call this characteristic). This method is too brutal in the lab, and too slow when you want to calculate $R_{\text{Th}}$ on paper.

In the lab, $I_{SC}$ could be too large for the health of your circuit (as in your fuse-blowing experience). You will soon learn a gentler way to get the same information.

On paper, if you are given the circuit diagram the fastest way to get $R_{\text{Th}}$ for a divider is always to take the parallel resistance of the several resistances that make up the divider (again assuming $R_{\text{source}}$ is ideal: zero $\Omega$). So, in the case you just examined:

$$V_{Th} = \frac{V_{in}}{R_{th}} = \frac{15V}{5k\Omega} = 3V$$

![Figure L1.7: $R_{Th} = \text{parallel resistances as seen from the circuit's output}](image)

1-5. Oscilloscope

We'll be using the oscilloscope ("scope") in virtually every lab from now on. If you run out of time today, you can learn to use the scope while doing the experiments of Lab 2. You will find Lab 2 easier, however, if today you can devote perhaps 20 minutes to meeting the scope.

Get familiar with scope and function generator (a box that puts out time-varying voltages: waveforms: things like sine waves, triangle waves and square waves) by generating a 1000 hertz (1kHz, 1000 cycles/sec) sine wave with the function generator and displaying it on the scope.

If both instruments seem to present you with a bewildering array of switches and knobs, don't blame yourself. These front-panels just are complicated. You will need several lab sessions to get fully used to them—and at term's end you may still not know all: it may be a long time before you find any occasion for use of the holdoff control, for example, or of single-shot triggering, if your scope offers this.
Play with the scope’s sweep and trigger controls. Specifically, try the following:

- The **vertical gain** switch. This controls “volts/div”; note that “div” or “division” refers to the **centimeter** marks, not to the tiny 0.2 cm marks;
- The **horizontal sweep speed** selector: **time** per division.

On this knob as on the vertical gain knob, make sure the switch is in its **CAL** position, not **VAR** or “variable.” Usually that means that you should turn a small center knob clockwise till you feel the switch **detent** click into place. If you don’t do this, you can’t trust any reading you take!

- The **trigger** controls. Don’t feel dumb if you have a hard time getting the scope to trigger properly. Triggering is by far the subtlest part of scope operation. When you think you have triggering under control, invite your partner to prove to you that you **don’t** have your partner randomize some of the scope controls, then see if you can regain a sensible display (don’t overdo it here!).

Beware the tempting so-called “normal” settings (usually labeled “NORM”). They rarely help, and instead cause much misery when misapplied. Think of “normal” here as short for **abnormal**! Save it for the rare occasion when you know you need it. “**AUTO**” is almost always the better choice.

Switch the function generator to square waves and use the scope to measure the “risetime” of the square wave (defined as time to pass from 10% to 90% of its full amplitude).

At first you may be inclined to despair, saying “Risetime? The square wave rises instantaneously.” The scope, properly applied, will show you this is not so.

---

**A suggestion on triggering:**

It’s a good idea to **watch** the edge that **triggers** the scope, rather than trigger on one event and watch another. If you watch the **trigger event**, you will find that you can sweep the scope fast without losing the display off the right side of the screen.

---

What comes out of the function generator’s **SYNC OUT** or **TTL** connector? Look at this on one channel while you watch a triangle or square wave on the other scope channel. To see how SYNC or TTL can be useful, try to trigger the scope on the peak of a sine wave without using these aids; then notice how entirely easy it is to trigger so when you **do** use SYNC or TTL to trigger the scope. (Triggering on a well-defined point in a waveform, such as peak or trough, is especially useful when you become interested in measuring a difference in **phase** between two waveforms; this you will do several times in the next lab.)

How about the terminal marked **CALIBRATOR** (or “CAL”) on the scope’s front panel? (We won’t ask you to use this signal yet; not until Lab 3 do we explain how a scope probe works, and how you “calibrate” it with this signal. For now, just note that this signal is available to you). **Postpone** using scope probes until you understand what is within one of these gadgets. A “10X” scope probe is **not** just a piece of coaxial cable.
Put an "offset" onto the signal, if your function generator permits, then see what the AC/DC switch (located near the scope inputs) does.

**Note on AC/DC switch:**
Common sense may seem to invite you to use the AC position most of the time: after all, aren’t these time-varying signals that you’re looking at "AC"—alternating current (in some sense)? **Eschew** this plausible error. The AC setting on the scope puts a capacitor in series with the scope input, and this can produce startling distortions of waveforms if you forget it is there. (See what a 50 Hz square wave looks like on AC, if you need convincing.) Furthermore, the AC setting washes away DC information, as you have seen: it hides from you the fact that a sine wave is sitting on a DC offset, for example. You don’t want to wash away information except when you choose to do so knowingly and purposefully. Once in a while you will want to look at a little sine with its DC level stripped away; but always you will want to know that this DC information has been made invisible.

Set the function generator to some frequency in the middle of its range, then try to make an accurate frequency measurement with the scope. (Directly, you are obliged to measure period, of course, not frequency.) You will do this operation hundreds of times during this course. Soon you will be good at it.

Trust the scope period readings; distrust the function generator frequency markings; these are useful only for very approximate guidance, on ordinary function generators.

Try looking at pulses, say 1μs wide, at 10kHz.

1-6. AC Voltage Divider
First spend a minute thinking about the following question: How would the analysis of the voltage divider be affected by an input voltage that changes with time (i.e., an input signal)? Now hook up the voltage divider from lab exercise 1-4, above, and see what it does to a 1kHz sine wave (use function generator and scope), comparing input and output signals.

*Figure L1.8: Voltage divider applied to a time-varying signal*

Explain in detail, to your own satisfaction, why the divider must act as it does.

If this question seems silly to you, you know either too much or too little.
Class 2: Capacitors and RC Circuits

Topics:

- new:
  - Capacitors: dynamic description
  - \( RC \) circuits

  ♦ Time domain:
    - step response
    - integrator, differentiator (approximate)

  ♦ Frequency domain: filters

Capacitors

Today things get a little more complicated, and more interesting, as we meet frequency-dependent circuits, relying on the capacitor to implement this new trick. Capacitors let us build circuits that “remember” their recent history. That ability allows us to make timing circuits (circuits that let \( this \) happen a predetermined time after \( that \) occurs); the most important of such circuits are oscillators—circuits that do this timing operation over and over, endlessly, in order to set the frequency of an output waveform. The capacitor’s memory also lets us make circuits that respond mostly to changes (differentiators) or mostly to averages (integrators), and—by far the most important—circuits that favor one frequency range over another (filters).

All of these circuit fragments will be useful within later, more complicated circuits. The filters, above all others, will be with us constantly as we meet other analog circuits. They are nearly as ubiquitous as the (resistive-) voltage divider that we met in the first class.

![Figure N2.1: The simplest capacitor configuration: sandwich](image)

This capacitor is drawn to look like a ham sandwich: metal plates are the bread, some dielectric is the ham (ceramic capacitors really are about as simple as this). More often, capacitors achieve large area (thus large capacitance) by doing something tricky, such as putting the dielectric between two thin layers of metal foil, then rolling the whole thing up like a roll of paper towel (mylar capacitors are built this way).

Text sec. 1.12

A static description of the way a capacitor behaves would say

\[
Q = CV
\]

where \( Q \) is total charge, \( C \) is the measure of how big the cap is (how much charge it can store at a given voltage: \( C = Q/V \)), and \( V \) is the voltage across the cap.
This statement just defines the notion of capacitance. It is a Physicist’s way of describing how a cap behaves, and rarely will we use it again. Instead, we use a dynamic description—a statement of how things change with time:

\[ I = C \frac{dV}{dt} \]

This is just the time derivative of the “static” description. \( C \) is constant with time; \( I \) is defined as the rate at which charge flows. This equation isn’t hard to grasp: it says ‘The bigger the current, the faster the cap’s voltage changes.’

Again, flowing water helps intuition: think of the cap (with one end grounded) as a tub that can hold charge:

![Diagram of a capacitor](image)

**Figure N2.2:** A cap with one end grounded works a lot like a tub of water

A tub of large diameter (cap) holds a lot of water (charge), for a given height (\( V \)). If you fill the tub through a thin straw (small \( I \)), the water level—\( V \)—will rise slowly; if you fill or drain through a fire hose (big \( I \)) the tub will fill (“charge”) or drain (“discharge”) quickly. A tub of large diameter (large capacitor) takes longer to fill or drain than a small tub. Self-evident, isn’t it?

**Time-domain Description**

*Text sec. 1.13*

Now let’s leave tubs of water, and anticipate what we will see when we watch the voltage on a cap change with time: when we look on a scope screen, as you will do in Lab 2.

**An easy case:** constant \( I \)

*Text sec. 1.15; see Fig. 1.43*

![Diagram of a ramp](image)

**Figure N2.3:** Easy case: constant \( I \) \( \rightarrow \) constant \( \frac{dV}{dt} \)

This tidy waveform, called a *ramp*, is useful, and you will come to recognize it as the signature of this circuit fragment: capacitor driven by constant current (or “current source”).
This arrangement is used to generate a triangle waveform, for example:

Compare Text sec.1.15, Fig. 1.42: ramp generator

Figure N2.4: How to use a cap to generate a triangle waveform: ramp up, ramp down

But the ramp waveform is relatively rare, because current sources are relatively rare. Much more common is the next case.

A harder case but more common: constant voltage source in series with a resistor

Figure N2.5: The more usual case: cap charged and discharged from a voltage source, through a series resistor

Here, the voltage on the cap approaches the applied voltage—but at a rate that diminishes toward zero as $V_{\text{cap}}$ approaches its destination. It starts out bravely, moving fast toward its $V_{\text{in}}$ (charging at 10 mA, in the example above, thus at 10V/ms); but as it gets nearer to its goal, it loses its nerve. By the time it is 1 volt away, it has slowed to 1/10 its starting rate.

(The cap behaves a lot like the hare in Xeno's paradox: remember him? Xeno teased his fellow-Athenians by asking a question something like this: 'If a hare keeps going halfway to the wall, then again halfway to the wall, does he ever get there?' (Xeno really had a hare chase a tortoise; but the electronic analog to the tortoise escapes us, so we'll simplify his problem.) Hares do bump their noses; capacitors don't: $V_{\text{cap}}$ never does reach $V_{\text{applied}}$ in an $RC$ circuit. But it will come as close as you want.)
Here's a fuller account:

Don't try to memorize these numbers, except two:

- in one RC (called "one time-constant") 63% of the way
- in five RCs, 99% of the way

If you need an exact solution to such a timing problem:

\[ V_{\text{cap}} = V_{\text{applied}} (1 - e^{-t/RC}) \]

In case you can't see at a glance what this equation is trying to tell you, look at \( e^{-t/RC} \) by itself:

- when \( t = RC \), this expression is \( 1/e \), or 0.37.
- when \( t \) = very large (\( \gg RC \)), this expression is tiny, and \( V_{\text{cap}} \approx V_{\text{applied}} \)

A tip to help you calculate time-constants:

\( \text{M}\Omega \) and \( \mu\text{F} \) give time-constant in seconds
\( \text{k}\Omega \) and \( \mu\text{F} \) give time-constant in milliseconds

In the case above, \( RC \) is the product of 1k and 1\( \mu\text{F} \): 1 ms.

**Integrators and Differentiators**

The very useful formula, \( I = C dV/dt \) will let us figure out when these two circuits perform pretty well as differentiator and integrator, respectively.
Let's, first, consider this simpler circuit:

![Circuit Diagram](image)

**Figure N2.8: Useless “differentiator”?**

The current that flows in the cap is proportional to \( dV_{in}/dt \): the circuit differentiates the input signal. But the circuit is pretty evidently useless. It gives us no way to measure that current. If we could devise a way to measure the current, we would have a differentiator.

Here's our earlier proposal, again. Does it work?

![Differentiator Diagram](image)

**Figure N2.9: Differentiator? — again**

Answer: Yes and No: Yes, to the extent that \( V_{cap} = V_{in} \) (and thus \( dV_{cap}/dt = dV_{in}/dt \)), because the circuit responds to \( dv/dt \) across the cap, whereas what interests us is \( dV_{in}/dt \)—that is, relative to ground.

So, the circuit errs to the extent that the output moves away from ground; but of course it must move away from ground to give us an output. This differentiator is compromised. So is the \( RC \) integrator, it turns out. When we meet operational amplifiers, we will manage to make nearly-ideal integrators, and pretty good differentiators.

The text puts this point this way:

*Text sec. 1.14*

for the differentiator:

“... choose \( R \) and \( C \) small enough so that \( dV/dt \ll dV_{in}/dt \)...”

*Text sec. 1.15*

for the integrator:

“...[make sure that] \( V_{out} \ll V_{in} \ldots \omega RC \gg 1 \).”

We can put this simply—perhaps crudely: assume a sine wave input. Then,

the \( RC \) differentiator (and integrator, too) works pretty well if it is **murdering** the signal (that is, attenuates it severely), so that \( V_{out} \) (and \( dV_{out} \)) is tiny: hardly moves away from ground.

It follows, along the way, that differentiator and integrator will impose a 90° phase shift on a sinusoidal input. This result, obvious here, should help you anticipate how \( RC \) circuits viewed as “filters” (below) will impose phase shifts.
Integrator

Assume $V_i$ is constant ...

$I$ not constant, to the extent $V_o$ moves

$dV_o$ should be constant.

But is it?

Figure N2.10: Integrator?—again

One can make a similar argument to explain the limitations of the $RC$ integrator. To keep things simple, imagine that you apply a step input; ask what waveform out you would like to see out, and what, therefore, you would like the current to do.

**RC Filters**

These are the most important application of capacitors. These circuits are just voltage dividers, essentially like the resistive dividers you have met already. The resistive dividers treated DC and time-varying signals alike. To some of you that seemed obvious and inevitable (and maybe you felt insulted by the exercise at the end of Lab 1 that asked you to confirm that AC was like DC to the divider). It happens because resistors can’t remember what happened even an instant ago. They’re little existentialists, living in the present. (We’re talking about ideal R’s, of course.)

**The impedance or reactance of a cap**

A cap’s impedance varies with frequency. (“Impedance” is the generalized form of what we called “resistance” for “resistors;” “reactance” is the term reserved for capacitors and inductors (the latter usually are coils of wire, often wound around an iron core)).

*Compare Text sec. 1.12*

It’s obvious that a cap cannot conduct a DC current: just recall what the cap’s insides look like: an insulator separating two plates. That takes care of the cap’s “impedance” at DC: clearly it’s infinite (or huge, anyway).

It is not obvious that a rapidly-varying voltage can pass “through” a capacitor, but that does happen. The Text explains this difficult notion at sec. 1.12, speaking of the current that passes through the cap. Here’s a second attempt to explain how a voltage signal passes through a cap, in the high-pass configuration. If you’re already happy with the result, skip this paragraph.

When we say the AC signal passes through, all we mean is that a wiggle on the left causes a wiggle of similar size on the right:

The wiggle makes it “across” the cap so long as there isn’t time for the voltage on the cap to change much before the wiggle has ended—before the voltage heads in the other direction. In other words, quick wiggles pass; slow wiggles don’t.
We can stop worrying about our intuition and state the expression for the cap's reactance:

\[ X_C = -j/\omega C = -j/2\pi f C \]

And once we have an expression for the impedance of the cap—an expression that shows it varying continuously with frequency—we can see how capacitors will perform in voltage dividers.

**RC Voltage dividers**

*Text sec. 1.18*

You know how a resistive divider works on a sine. How would you expect a divider made of capacitors to treat a time-varying signal?

![Diagram of RC voltage dividers]

If this case worries you, good: you're probably worrying about phase shifts. Turns out they cause no trouble here: output is in phase with input. (If you can handle the complex notation, write \( X_C = -j/\omega C \) , and you'll see the j's wash out.)

But what happens in the combined case, where the divider is made up of both R and C? This turns out to be an extremely important case.

*Text sec. 1.19*

This problem is harder, but still fits the voltage-divider model. Let's generalize that model a bit:

![Generalized voltage divider diagram]

The behavior of these voltage dividers—which we call filters when we speak of them in frequency terms, because each favors either high or low frequencies—is easy to describe:

1. See what the filter does at the two frequency extremes. (This looking at extremes is a useful trick; you will soon use it again to find the filters' worst-case \( Z_{in} \) and \( Z_{out} \).)

![Filter frequency response diagram]

At \( f = 0 \): what fraction out? At \( f = \) very high: what fraction out?
2. Determine where the output “turns the corner” (corner defined arbitrarily\(^1\)) as the frequency where the output is 3dB less than the input (always called just “the 3 dB point”; “minus” understood).

Knowing the endpoints, which tell us whether the filter is high-pass or low-pass, and knowing the 3dB point, we can draw the full frequency-response curve:

The “3dB point,” the frequency where the filter “turns the corner” is

\[
f_{3\text{dB}} = \frac{1}{2\pi RC}
\]

_Beware_ the more elegant formulation that uses \(\omega\):

\[
\omega_{3\text{dB}} = \frac{1}{RC}.
\]

That is tidy, but is very likely to give you answers off by about a factor of 6, since you will be measuring period and its inverse in the lab: frequency in hertz (or “cycles-per-second,” as it used to be called), not in radians.

**Two asides:**

**Caution!**

Do not confuse these _frequency-domain_ pictures with the earlier _RC_ step-response picture, (which speaks in the _time-domain_).

Not only do the curves look vaguely similar. To make things worse, details here seem tailor-made to deceive you:

- **Step response**: in the _time RC_ (time-constant), \(V_{\text{cap}}\) moves to about 0.6 of the applied step voltage (this is \(1 - 1/e\)).
- **Frequency domain**: at \(f_{3\text{dB}}\), a frequency determined by \(RC\), the filter’s \(V_{\text{out}}/V_{\text{in}}\) is about 0.7 (this is \(1/\sqrt{2}\))

Don’t fall into this trap.

_A note re Log Plots_

You may wonder why the curves we have drawn, the curve in Fig. 1.59, and those you see on the scope screen (when you “sweep” the input frequency) don’t look like the tidier curves shown in most books that treat frequency response, or like the curves in Chapter 5 of

---

1. Well, not quite arbitrarily: a signal reduced by 3dB delivers half its original power.
the Text. Our curves trickle off toward zero, in the low-pass configuration, whereas these other curves seem to fall smoothly and promptly to zero. This is an effect of the logarithmic compression of the axes on the usual graph. Our plots are linear; the usual plot ("Bode plot") is log-log:

![Log-log plots](image)

**Figure N2.17: Linear versus log-log frequency-response plots contrasted**

### Input and output impedance of an RC Circuit

If filter A is to drive filter B—or anything else, in fact—it must conform to our 10X rule of thumb, which we discussed last time, when we were considering only resistive circuits. The same reasons prevail, but here they are more urgent: if we don’t follow this rule, not only will signals get attenuated; frequency response also is likely to be altered.

But to enforce our rule of thumb, we need to know $Z_{in}$ and $Z_{out}$ for the filters. At first glance, the problem looks nasty. What is $Z_{out}$ for the low-pass filter, for example? A novice would give you an answer that’s much more complicated than necessary. He might say,

$$Z_{out} = X_C \parallel R = -j/\omega C \cdot R / (-j/\omega C + R)$$

Yow! And then this expression doesn’t really give an answer: it tells us that the answer is frequency-dependent.

We cheerfully sidestep this hard work, by considering only worst case values. We ask, ‘How bad can things get?’

We ask, ‘How bad can $Z_{in}$ get?’ And that means, ‘How low can it get?’

We ask, ‘How bad can $Z_{out}$ get?’ And that means, ‘How high can it get?’

This trick delivers a stunningly-easy answer: the answer is always just $R$! Here’s the argument for a low-pass, for example:

worst $Z_{in}$: cap looks like a short: $Z_{in} = R$

(this happens at highest frequencies)

worst $Z_{out}$: cap doesn’t help at all; we look through to the source, and see only $R$: $Z_{out} = R$

(this happens at lowest frequencies)

**Figure N2.18: Worst-case $Z_{in}$ and $Z_{out}$ for RC filter reduces to just $R$**

Now you can string together $RC$ circuits just as you could string together voltage dividers, without worrying about interaction among them.
Phase Shift

You already know roughly what to expect: the differentiator and integrator showed you phase shifts of 90°, and did this when they were severely attenuating a sine-wave. You need to beware the misconception that because a circuit has a cap in it, you should expect to see a 90° shift (or even just noticeable shift). That is not so. You need an intuitive sense of when phase shifting occurs, and of roughly its magnitude. You rarely will need to calculate the amount of shift.

Here is a start: a rough account of phase shift in RC circuits:

If the amplitude out is close to amplitude in, you will see little or no phase shift. If the output is much attenuated, you will see considerable shift (90° is maximum)

And here are curves saying the same thing:

Why does this happen? Here's an attempt to rationalize the result:

- voltages in R and C are 90° out of phase, as you know.
- the sum of the voltages across R and C must equal, at every instant, \( V_{in} \).
- as frequency changes, R and C share the total \( V_{in} \) differently, and this alters the phase of \( V_{out} \) relative to \( V_{in} \).

Consider a low-pass, for example: if a lot of the total voltage, \( V_{in} \), appears across the cap, then the phase of the input voltage (which appears across the \( RC \) series combination) will be close to the phase of the output voltage, which is taken across the cap alone. In other words, R plays a small part: \( V_{out} \) is about the same as \( V_{in} \), in both amplitude and phase. Have we merely restated our earlier proposition? It almost seems so.
But let's try a drawing:

![Diagram of RC Circuit]

Figure N2.20: R and C sharing input voltage differently at two different frequencies

Now let's try another aid to an intuitive understanding of phase shift: phasors.

### Phasor Diagrams

These diagrams let you compare phase and amplitude of input and output of circuits that shift phases (circuits including C's and L's). They make the performance graphic, and allow you to get approximate results by use of geometry rather than by explicit manipulation of complex quantities.

The diagram uses axes that represent resistor-like (“real”) impedances on the horizontal axis, and capacitor or inductor-like impedances (“imaginary”—but don’t let that strange name scare you; for our purposes it only means that voltages across such elements are 90° out of phase with voltages across the resistors). This plot is known by the extra-frightening name, “complex plane” (with nasty overtones, to the untrained ear, of ‘too-complicated-for-you plane’!). But don’t lose heart. It’s all very easy to understand and use. Even better, you don’t need to understand phasors, if you don’t want to. We use them rarely in the course, and always could use direct manipulation of the complex quantities instead. Phasors are meant to make you feel better. If they don’t, forget them.

![Phasor Diagram]

Figure N2.21: Phasor diagram: “complex plane,” showing an RC at f_{3dB}

The diagram above shows an RC filter at its 3dB point, where, as you can see, the magnitude of the impedance of C is the same as that of R. The arrows, or vectors, show phase as well as amplitude (notice that this is the amplitude of the waveform: the peak value, not a voltage varying with time); they point at right angles so as to say that the voltages in R and C are 90° out of phase.

“Voltages?,” you may be protesting, “but you said these arrows represent impedances.” True. But in a series circuit the voltages are proportional to the impedances, so this use of the figure is fair.
The total impedance that R and C present to the signal source is not 2R, but is the vector sum: it’s the length of the hypotenuse, $R\sqrt{2}$. And from this diagram we now can read two familiar truths about how an $RC$ filter behaves at its 3dB point:

- the amplitude of the output is down 3dB: down to $1/\sqrt{2}$: the length of either the R or the C vector, relative to the hypotenuse.
- the output is shifted 45° relative to the input: R or C vectors form an angle of 45° with the hypotenuse, which represents the phase of the input voltage.

So far, we’re only restating what you know. But to get some additional information out of the diagram, try doubling the input frequency several times in succession, and watch what happens:

each time, the length of the $X_c$ vector is cut to half what it was.

First doubling of frequency:

\[
\begin{align*}
\text{Figure N2.22: } & \text{RC after a doubling of frequency, relative to the previous diagram} \\
R & \quad X_c = \text{half what it was: } R/2
\end{align*}
\]

The first doubling also affects the length of the hypotenuse substantially, too, however; so the amplitude relative to input is not cut quite so much as 50% (6dB). You can see that the output is a good deal more attenuated, however, and also that phase shift has increased a good deal.

Second doubling of frequency

\[
\begin{align*}
\text{Figure N2.23: } & \text{RC after a doubling of frequency, relative to the previous diagram} \\
R & \quad X_c = \text{small (<<15kΩ)}
\end{align*}
\]

This time, the length of the hypotenuse is changed less, so the output shrinks nearly as the $X_c$ vector shrinks: nearly 50%. Here, we are getting close to the -6dB/octave slope of the filter’s rolloff curve. Meanwhile, the phase shift between output and input is increasing, too—approaching the limit of 90°.

We’ve been assuming a low-pass. If you switch assumptions, and ask what these diagrams show happening to the output of a high-pass, you find all the information is there for you to extract. No surprise, there; but perhaps satisfying to notice.
**LC circuit on phasor diagram**

Finally, let's look at an LC trap circuit on a phasor diagram.

![Phasor diagram of LC trap circuit](image)

This is less familiar, but pleasing because it reveals the curious fact—which you will see in Lab 3 when you watch a similar (parallel) LC circuit—that the LC combination looks sometimes like L, sometimes C, showing its characteristic phase shift—and at resonance, shows no phase shift at all. We'll talk about LC's next time; but for the moment, see if you can enjoy how concisely this phasor diagram describes this behavior of the circuit (actually a trio of diagrams appears here, representing what happens at three sample frequencies).

To check that these LC diagrams make sense, you may want to take a look at what the old voltage-divider equation tells you ought to happen:

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{Z_{\text{combination}}}{Z_{\text{combination}} + R}
\]

But

\[
Z_{\text{combination}} = -\frac{j}{\omega C} + j\omega L.
\]

And at some frequency—where the magnitudes of the expressions on the right side of that last equation are equal—the sum is zero, because of the opposite signs. Away from this magic frequency (the "resonant frequency"), either cap or inductor dominates. Can you see all this on the phasor diagram?

**Better Filters**

Having looked hard at RC filters, maybe we should remind of the point that the last exercise in Lab 2 means to make: RC’s make extremely useful filters, but if you need a better filter, you can make one, either with an LC combination, as in that circuit, or with operational amplifiers cleverly mimicking such an LC circuit (this topic is treated in Chapter 5; we will not build such a circuit), or with a clever circuit called a ‘switched-capacitor’ filter, a circuit that you will get a chance to try, in Lab 11, and again in Lab 21. Here is a sketch (based on a scope photo) comparing the output of an ordinary RC low-pass against a 5-pole Butterworth low-pass, like the one you will build at the end of Lab 2.
"5-pole" 4C Butterworth lowpass

RC lowpass

output envelope

frequency (linear sweep) 100kHz

Figure N2.26: Simple RC low-pass contrasted with 5-pole Butterworth low-pass

("5-pole" is a fancy way to say that it something like, 'It rolls off the way 5 simple RC's in series would roll off"—it's five times as steep as the plain RC. But this nice filter works a whole lot better than an actual string of 5 RC's.)

Not only is the roll-off of the Butterworth much more abrupt than the simple RC's, but also the "passband" looks much flatter: the fancy filter does a better job of passing. The poor old RC looks sickly next to the Butterworth, doesn't it?

Nevertheless, we will use RC's nearly always. Nearly always, they are good enough. It would not be in the spirit of this course to pine after a more beautiful transfer function. We want circuits that work, and in most applications the plain old RC passes that test.
Two worked exercises:

1. filter to keep "signal" and reject "noise"
2. bandpass filter

1. Filter to keep "signal" and reject "noise"

Problem:

*Filter to remove fuzz:*

Suppose you are faced with a signal that looks like this: a signal of moderate frequency, polluted with some fuzz

![Figure X2.1: Signal With Fuzz Added](image)

1. Draw a skeleton circuit (no parts values, yet) that will keep most of the good signal, clearing away the fuzz.
2. Now choose some values:
   a. If the *load* has value 100k, choose R for your circuit.
   b. Choose $f_{3\text{dB}}$, explaining your choice briefly.
   c. Choose C to achieve the $f_{3\text{dB}}$ that you chose.
   d. By about how much does your filter attenuate the noise "fuzz"?

What is the circuit’s input impedance—

a. at very low frequencies?
b. at very high frequencies?
c. at $f_{3\text{dB}}$?

3. What happens to the circuit output if the load has resistance 10k rather than 100k?
A Solution:

1. Skeleton Circuit

You need to decide whether you want a low-pass or high-pass, since the signal and noise are distinguishable by their frequencies (and are far enough apart so that you can hope to get one without the other, using the simple filters we have just learned about). Since we have called the lower frequency "good" or "signal," we need a low-pass:

![Figure X2.2: Skeleton: just a low-pass filter](image)

2. Choose $R$, given the load

This dependence of $R$ upon load follows from the observation that $R$ of an RC filter defines the worst-case input and output impedance of the filter (see Class 2 notes). We want that output impedance low relative to the load's impedance; our rule of thumb says that 'low' means low by a factor of 10. So, we want $R \leq R_{\text{load}}/10$. In this case, that means $R$ should be $\leq 10k$. Let's use 10k.

3. Choose $f_{3\text{dB}}$

This is the only part of the problem that is not entirely mechanical. We know we want to pass the low and attenuate the high, but does that mean put $f_{3\text{dB}}$ halfway between good and bad? Does it mean put it close to good? ...Close to bad? Should both good and bad be on a steeply-falling slope of the filter's response curve?

![Figure X2.3: Where should we put $f_{3\text{dB}}$? Some possibilities](image)

Assuming that our goal is to achieve a large ratio of good to bad signal, then we should not put $f_{3\text{dB}}$ close to the noise: if we did we would not do a good job of attenuating the bad. Halfway between is only a little better. Close to signal is the best idea: we will then attenuate the bad as much as possible while keeping the good, almost untouched.

An alert person might notice that the greatest relative preference for good over bad comes when both are on the steepest part of the curve showing frequency response: in other words, put $f_{3\text{dB}}$ so low that both good and bad are attenuated. This is a clever answer—but wrong, in most settings.

The trouble with that answer is that it assumes that the signal is a single frequency. Ordinarily the signal includes a range of frequencies, and it would be very bad to choose $f_{3\text{dB}}$ somewhere within that range: the filter would distort signals.

So, let's put $f_{3\text{dB}}$ at $2f_{\text{signal}}$: around 2kHz. This gives us 89% of the original signal amplitude (as you can confirm if you like with a phasor diagram or direct calculation). At
the same time we should be able to attenuate the 16kHz noise a good deal (we’ll see in a moment how much).

4. Choose $C$ to achieve the $f_{3dB}$ that you want

This is entirely mechanical: use the formula for the 3dB point:

$$f_{3dB} = \frac{1}{(2\pi RC)} \Rightarrow C = \frac{1}{(2\pi f_{3dB} R)} = \frac{1}{(6\times2\times10^3 \times 10 \times 10^3)} = \frac{1}{120 \times 10^6} \approx 0.008 \times 10^{-6} \text{ F}$$

We might as well use a 0.01 µF cap (“cap” ≡ capacitor). It will put our $f_{3dB}$ about 25% low—1.6KHz; but our choice was a rough estimate anyway.

5. By about how much does your filter attenuate the noise (“fuzz”)?

The quick way to get this answer is to count octaves or decades between $f_{3dB}$ and the noise. $f_{3dB}$ is 2 kHz; the fuzz is around 16kHz: $8 \cdot f_{3dB}$.

*Count octaves:* we could also say that the frequency is doubled three times ($= 2^3$) between $f_{3dB}$ and the noise frequency. *Roughly,* that means that the fuzz amplitude is cut in half the same number of times: down to $1/(2^3)$: $1/8$.

This is only an approximate answer because a) near $f_{3dB}$ the curve has not yet reached its terminal steepness of $-6$ dB/octave; b) on the other hand, even at $f_{3dB}$, some attenuation occurs. But let’s take our rough answer. (It happens that our rough answer is better than it deserves to be: we called $V_{out}/V_{in}$ 0.125; the more exact answer is 0.124.)

6. What happens to the circuit output if the load has resistance 10k rather than 100k?

Here’s a picture of such loading.

![Figure X2.4: Overloaded filter](image)

If you have gotten used to Thevenin models, then you can see how to make this circuit look more familiar:

![Figure X2.5: Loaded circuit, redrawn](image)

The amplitude is down; but, worse, $f_{3dB}$ has changed: it has doubled. You will find a plot showing this effect at the start of the class notes for next time: Class 3.
7. What is the circuit's input impedance—

1. at very low frequencies?
   Answer: very large: the cap shows a high impedance; the signal source sees only the load—which is assumed very high impedance (high enough so we can neglect it as we think about the filter's performance)

2. at very high frequencies?
   Answer: $R$: The cap impedance falls toward zero—but $R$ puts a lower limit on the input impedance.

3. at $f_{3\text{dB}}$?

   This is easy if you are willing to use phasors, a nuisance to calculate, otherwise. If you recall that the magnitude of $X_C = R$ at $f_{3\text{dB}}$, and if you accept the notion that the voltages across $R$ and $C$ are $90^\circ$ out of phase, so that they can be drawn at right angles to each other on a phasor diagram, then you get the phasor diagram you saw in the class notes (fig. N2.21):

   ![Phasor Diagram](image)

   **Figure X2.6:** Phasor diagram showing an RC filter at its 3dB point

   —then you can use a geometric argument to show that the hypotenuse—proportional to $Z_{in}$—is $R\sqrt{2}$.

2. Bandpass

   *Text exercise AE-6 (Chapter 1)*

   **Problem:**

   **Bandpass filter**

   Design a bandpass filter to pass signals between about 1.6 kHz and 8kHz (you may use these as 3dB frequencies).

   Assume that the next stage, which your bandpass filter is to drive, has an input impedance of 1 M ohms.
Once you recognize that to get this frequency response from the filter you need to put high-pass and low-pass in series, the task is mechanical. You can put the two filters in either order. Now we need to choose $R$ values, because these will determine worst-case impedances for the two filter stages. The later filter must show $Z_{out}$ low relative to the load, which is $1\,\text{M}\Omega$; the earlier filter must show $Z_{out}$ low relative to $Z_{in}$ of the second filter stage.

So, let the second-stage $R = 100\,\text{k}$; the first-stage $R = 10\,\text{k}$:

### Choosing C’s

Here the only hard part is to get the filter’s right: it’s hard to say to oneself, ‘The high-pass filter has the lower $f_{3\text{dB}}$’; but that is correct. Here are the calculations: notice that we try to keep things in engineering notation—writing ‘$10^5$’ rather than ‘$10^4$.’ This form looks clumsy, but rewards you by delivering answers in standard units. It also helps you scan for nonsense in your formulation of the problem: it is easier to see that ‘$10^3$’ is a good translation for ‘$10\,\text{k}$’ than it is to see that, say, ‘$10^5$’ is not a good translation.

![Diagram of RC circuit](image)

**Figure X2.8:** Calculating $C$ values: a plug for engineering notation
A Note on Reading Capacitor Values

Why you may need this note

Most students learn pretty fast to read resistor values. They tend to have more trouble finding, say, a 100 pF capacitor.

That's not their fault. They have trouble, as you will agree when you have finished reading this note, because the cap manufacturers don't want them to be able to read cap values. ("Cap" is shorthand for "capacitor," as you probably know.) The cap markings have been designed by an international committee to be nearly unintelligible. With a few hints, however, you can learn to read cap markings, despite the manufacturers' efforts. Here are our hints:

Big Caps: electrolytics

These are easy to read, because there is room to write the value on the cap, including units. You need only have the common sense to assume that

```
+ 500µF
```

Figure CAP.1: A big cap is labeled intelligibly

means 500 micro farads, not what it would mean if you took the capital M seriously.

All of these big caps are polarized, incidentally. That means the capacitor's innards are not symmetrical, and that you may destroy the cap if you apply the wrong polarity to the terminals: the terminal marked + must be at least as positive as the other terminal. (Sometimes, violating this rule will generate gas that makes the cap blow up; more often, you will find the cap internally shorted, after a while. Often, you could get away with violating this rule, at low voltages. But don't try.)

Small Caps

As the caps get smaller, the difficulty in reading their markings gets steadily worse.

Tantalum

These are the silver colored cylinders. They are polarized: a + mark and a metal nipple mark the positive end. Their markings may say something like

```
+ 4.7µF
```

Figure CAP.2: Tantalum cap

That means pretty much what it says, if you know that the "R" marks the decimal place: it's a 4.7 µF cap.

The same cap is also marked,

```
+ 475K
```

Figure CAP.3: Tantalum cap: second marking scheme on same part

Here you meet your first challenge, but also the first appearance of an orderly scheme for labeling caps, a scheme that would be helpful if it were used more widely.

The challenge is to resist the plausible assumption that "k" means "kilo." It does not; it is not a unit marking, but a tolerance notation (it means ± 10%). (Wasn't that nasty of the labelers to choose "K"? Guess what's another favorite letter for tolerance. That's right: M. Pretty mean!)
The orderly labeling here mimics the resistor codes: 475 means 
\[47 \times \text{ten to the fifth}.\]

What units?

10\(^5\) what? 10\(^5\) of something small. You will meet this question repeatedly, and you must resolve it by relying on a few observations:

1. The only units commonly used in this country are
   - microfarads: \(10^{-6}\) Farad
   - picofarads: \(10^{-12}\) Farad

   (You should, therefore, avoid using “mF” and “nF,” yourself.)

   A Farad is a huge unit. The biggest cap you will use in this course is 500 \(\mu\)F. That cap is physically large. (We do keep a 1F cap around, but only for our freak show.) So, if you find a little cap labeled “680,” you know it’s 680 \(\mu\)F.

2. A picofarad is a tiny unit. You will not see a cap as small as 1 \(\mu\)F in this course. So, if you find a cap claiming that it is a fraction of some unstated unit—say, “.01”—the unit is \(\mu\)F’s: “.01” means 0.01 \(\mu\)F.

3. \textit{Beware} the wrong assumption that a \textit{picofarad} is only a bit smaller than a microfarad. A \(p\)F is \textit{not} \(10^{-9}\) F (\(10^{-3}\ \mu\)F); instead, it is \(10^{-12}\)F: a \textit{million} times smaller than a microfarad.

So, we conclude, this cap labeled “475” must be \(4.7\times10^6\ \text{picofarads}\). That, you will recognize, is a roundabout way to say

\[4.7 \times 10^{-6} \text{ F}\]

We knew that was the answer, before we started this last decoding effort. This way of labeling is indeed roundabout, but at least it is unambiguous. It would be nice to see it used more widely. You will see another example of this \textit{exponential} labeling in the case of the CK05 ceramics, below.

Mylar

These are yellow cylinders, pretty clearly marked. .01M is just 0.01 \(\mu\)F, of course; and .1MFD is \textit{not} a tenth of a megafarad. These caps are not polarized; the black band marks the outer end of the foil winding. We don’t worry about that fine point. Orient them at random in your circuits.

Because they are long \textit{coils} of metal foil (separated by a thing dielectric—the “mylar” that gives them their name), mylar caps must betray their coil-like construction at very high frequencies: that is, they begin to fail as capacitors, behaving instead like inductors, blocking the very high frequencies they ought to pass. Ceramics (below) do better in this respect, though they are poor in other characteristics.
Ceramic

These are little orange pancakes. Because of this shape (in contrast to the coil format hidden within the tubular shape of mylars) they act like capacitors even at high frequencies. The trick, in reading these, is to reject the markings that can't be units:

Disc

The trick, in reading these, is to reject the markings that can't be units:

*Z5U:* Not a unit marking: cap type .02M, 560M That’s it: the M is a tolerance marking, as you know (± 20%); not a unit
Common sense tells you units:
“.02?” microfarads. “560?” picofarads.

*1kV:* Not a unit marking. Instead, this means—as you would guess—that the cap can stand 1000 volts.

CK05

These are little boxes, with their leads 0.2“ apart. They are handy, therefore, for insertion into a printed circuit.

101k: This is the neat resistor-like marking. This one is 100 pF.

Tolerance Codes

Just to be thorough—and because this information is hard to come by—let’s list all the tolerance codes. These apply to both capacitors and resistors; the tight tolerances are relevant only to resistors; the strangely-asymmetric tolerance is used only for capacitors.

<table>
<thead>
<tr>
<th>Tolerance Code</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>+80%, −20% (for big filter capacitors, where you are assumed to have asymmetric worries: too small a cap allows excessive “ripple;” more on this in Lab 3 and Notes 3)</td>
</tr>
<tr>
<td>M</td>
<td>± 20%</td>
</tr>
<tr>
<td>K</td>
<td>± 10%</td>
</tr>
<tr>
<td>J</td>
<td>± 5%</td>
</tr>
<tr>
<td>G</td>
<td>± 2%</td>
</tr>
<tr>
<td>F</td>
<td>± 1%</td>
</tr>
<tr>
<td>D</td>
<td>± 0.5%</td>
</tr>
<tr>
<td>C</td>
<td>± 0.25%</td>
</tr>
<tr>
<td>B</td>
<td>± 0.1%</td>
</tr>
<tr>
<td>A</td>
<td>± 0.05%</td>
</tr>
<tr>
<td>Z</td>
<td>± 0.025 (precision resistors; context will show the asymmetric cap tolerance “Z” makes no sense here)</td>
</tr>
<tr>
<td>N</td>
<td>± 0.02%</td>
</tr>
</tbody>
</table>

Figure CAP.7: Tolerance codes
# Lab 2: Capacitors

**Reading:**
Chapter 1.12 – 1.21, pp. 20-40;  
Appendix B (if you need it).  

**Warning:** This is by far the most mathematical portion of the course. Don’t panic. Even if you don’t understand the math, you’ll be able to understand the rest of the book.  

Student Manual note on reading capacitor values (Manual, pp. 51-53)  

**Problems:**
Problems in text.  
Additional Exercises 3-6.

## 2-1 RC Circuit

![RC Circuit Diagram](attachment:image.png)  

Figure L2.1: RC Circuit: step response

Verify that the RC circuit behaves in the time domain as described in Text sec. 1.13. In particular, construct the circuit above. Use a mylar capacitor (yellow tubular package, with one lead sticking out each end: “axial leads”). Drive the circuit with a 500Hz square wave, and look at the output. Be sure to use the scope’s DC input setting. (Remember the warning about the AC setting, last time?)

Measure the time constant by determining the time for the output to drop to 37%. Does it equal the product \( RC \)?

**Suggestion:** The percent markings over at the left edge of the scope screen are made-to-order for this task: put the foot of the square wave on 0%, the top on 100%. Then crank up the sweep rate so that you use most of the screen for the fall from 100% to around 37%.

Measure the time to climb from 0% to 63%. Is it the same as the time to fall to 37%? (If not, something is amiss in your way of taking these readings!)

Try varying the frequency of the square wave.
2-2 Differentiator

Construct the $RC$ differentiator shown above. Drive it with a square wave at 100kHz, using the function generator with its attenuator set to 20dB. Does the output make sense? Try a 100kHz triangle wave. Try a sine.

**Input Impedance**

Here’s your first chance to try getting used to quick worst-case impedance calculations, rather than exact and frequency-dependent calculations (which often are almost useless).

- What is the impedance presented to the signal generator by the circuit (assume no load at the circuit’s output) at $f = 0$?
- At infinite frequency?

Questions like this become important when the signal source is less ideal than the function generators you are using.

2-3 Integrator

Construct the integrator shown above. Drive it with a 100kHz square wave at maximum output level (attenuator set at 0dB).

What is the input impedance at dc? At infinite frequency? Drive it with a triangle wave; what is the output waveform called? (Doesn’t this circuit seem clever? Doesn’t it remember its elementary calculus better than you do—or at least faster?)

To expose this as only an approximate or conditional integrator, try dropping the input frequency. Are we violating the stated condition (sec. 1.15):

$$v_{out} \ll v_{in}?$$

The differentiator is similarly approximate, and fails unless (sec. 1.14):

$$\frac{dV_{out}}{dt} \ll \frac{dV_{in}}{dt}$$

In a differentiator, $RC$ too large tends to violate this restriction. If you are extra zealous you may want to look again at the differentiator of experiment 2-2, but this time increasing $RC$ by a factor of, say, 1000. The “derivative” of the square wave gets ugly, and this will not surprise you; the derivative of the triangle looks odd in a less obvious way.

When we meet operational amplifiers in Chapter 3, we will see how to make “perfect” differentiators and integrators—those that let us lift the restrictions we have imposed on these $RC$ versions.
2-4 Low-pass Filter

[Figure L2.4: RC low-pass filter]

Construct the low-pass filter shown above.

Aside: "Integrator" versus "Low-pass Filter"

'Wait a minute!', you may be protesting, 'Didn’t I just build this circuit?' Yes, you did. Then why do it again? We expect that you will gradually divine the answer to that question as you work your way through this experiment. One of the two experiments might be called a special case of the other. When you finish, try to determine which is which.

What do you calculate to be the filter’s −3dB frequency? Drive the circuit with a sine wave, sweeping over a large frequency range, to observe its low-pass property; the 1kHz and 10kHz ranges should be most useful.

Find \( f_{3\text{dB}} \) experimentally: measure the frequency at which the filter attenuates by 3dB (\( v_{out} \) down to 70.7% of full amplitude).

Note: henceforth we will refer to "the 3dB point" and "\( f_{3\text{dB}} \)," henceforth, not to the minus 3dB point, or \( f_{-3\text{dB}} \). This usage is confusing but conventional; you might as well start getting used to it.

What is the limiting phase shift, both at very low frequencies and at very high frequencies?

Suggestion:

As you measure phase shift, use the function generator’s SYNC or TTL output to drive the scope’s External Trigger. That will define the input phase cleanly. Then use the scope’s variable sweep rate so as to make a full period of the input waveform use exactly 8 divisions (or centimeters). The output signal, viewed at the same time, should reveal its phase shift readily.

Check to see if the low-pass filter attenuates 6dB/octave for frequencies well above the −3dB point; in particular, measure the output at 10 and 20 times \( f_{3\text{dB}} \). While you’re at it, look at phase shift vs frequency: What is the phase shift for

\[
\begin{align*}
f &< f_{3\text{dB}}, \\
f &= f_{3\text{dB}}, \\
f &> f_{3\text{dB}}?
\end{align*}
\]
Finally, measure the attenuation at $f = 2f_{3\text{dB}}$ and write down the attenuation figures at $f = 2f_{3\text{dB}}$, $f = 4f_{3\text{dB}}$ and $f = 10f_{3\text{dB}}$ for later use: in section 2-9, below, we will compare this filter against one that shows a steeper rolloff.

### Sweeping Frequencies

This circuit is a good one to look at with the function generator's sweep feature. This will let your scope draw you a plot of amplitude versus frequency instead of amplitude versus time as usual. If you have a little extra time, we recommend this exercise. If you feel pressed for time, save this task for next time, when the LC resonant circuit offers you another good target for sweeping.

To generate such a display of $v_{\text{out}}$ versus frequency, let the generator's ramp output drive the scope's horizontal deflection, with the scope in "X-Y" mode: in X-Y, the scope ignores its internal horizontal deflection ramp (or "timebase") and instead lets the input labeled "X" determine the spot's horizontal position.

The function generator's ramp time control now will determine sweep rate. Keep the ramp slow: a slow ramp produces a scope image that is annoyingly intermittent, but gives the truest, prettiest picture, since the slow ramp allows more cycles in a given frequency range than are permitted by a faster ramp.

### 2-5 High-pass Filter

![Figure L2.5: RC high-pass filter](image)

Construct a high-pass filter with the components that you used for the low-pass. Where is this circuit's 3dB point? Check out how the circuit treats sine waves: Check to see if the output amplitude at low frequencies (well below the -3dB point) is proportional to frequency. What is the limiting phase shift, both at very low frequencies and at very high frequencies?

### 2-6 Filter Application I: Garbage Detector

![Figure L2.6: High-pass filter applied to the 60Hz ac power](image)

The circuit above will let you see the "garbage" on the 110-volt power line. First look at the output of the transformer, at A. It should look more or less like a classical sine wave. (The transformer, incidentally, serves two purposes — it reduces the 110Vac to a more...
reasonable 6.3V, and it “isolates” the circuit we’re working on from the potentially lethal power line voltage)

To see glitches and wiggles, look at B, the output of the high-pass filter. All kinds of interesting stuff should appear, some of it curiously time-dependent. What is the filter’s attenuation at 60Hz? (No complex arithmetic necessary. Hint: count octaves, or use the fact—which you confirmed just above—that amplitude grows linearly with frequency, well below f_{3dB}.)

2-7 Filter Application II: Selecting signal from signal plus noise

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Composite signal, consisting of two sine waves. (The 1k resistor protects the function generator in case the composite output is accidentally shorted to ground)}
\end{figure}

Now we will try using high-pass and then low-pass filters to prefer one frequency range or the other in a composite signal, formed as shown in the figure above. The transformer adds a large 60Hz sine wave (peak value about 10 volts) to the output of the function generator.

Run this composite signal through the high-pass filter shown below.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{High-pass filter}
\end{figure}

Look at the resulting signal. Calculate the filter’s 3dB point.

Is the attenuation of the 60Hz waveform about what you would expect? Note that this time the 60Hz is considered “noise.” (In fact, as you will gather gradually, this is the most common and troublesome source of noise in the lab.)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3.png}
\caption{Low-pass filter}
\end{figure}

Now run the composite signal through the low-pass filter shown above, instead of running it through the high-pass. Look at the resulting signal. Calculate this circuit’s 3dB point. Why were the 3dB frequencies chosen where they were?
2-8 Blocking Capacitor

Capacitors are used very often to "block" dc while coupling an ac signal. The circuit above does this. You can think of it as a high-pass filter, with all signals of interest well above the 3dB point. This way of describing what the circuit does is subtly different from the way one usually speaks of a filter: the filter's job is to prefer a range of good frequencies, while attenuating another range, the "noise." The blocking capacitor is doing something different; its mission is not to attenuate evil low frequencies; its mission, instead, is simply to block the irrelevant dc. (Once again the alert and skeptical student may be objecting, 'I've built this circuit twice before. You keep re-naming it and asking me to build it again: "differentiator," "high-pass," and now "blocking capacitor." Again we must answer that it is the same circuit, but it is applied differently from the others.)

To see this application in action, wire up the circuitry labeled "A," above: the left side of the circuit.

Drive it with the function generator, and look at the output on the scope, dc coupled, as usual. The circuit lets the ac signal "ride" on +5 volts. Next, add the circuitry labeled "B," above (another blocking capacitor), and observe the signal back at ground. What is the low frequency limit for this blocking circuit?

This circuit fragment you have just built looks quite useless.
As it stands, it is useless, since it gets you back where you started, doing nothing useful between. Next time you will see applications for this kind of circuit. The difference will be that you will do something useful to the signal after getting it to ride on, say, +5 volts.

2-9 LC Filter

It is possible to construct filters (high pass, low pass, etc.) with a frequency response that is far more abrupt than the response of the simple RC filters you have been building, by combining inductors with capacitors, or by using amplifiers or electronic switching (the latter two types are called "active filters"—see Chapter 5 in the text—or "switched capacitor" types—see sec. 5.11, p. 281ff).

To get a taste of what can be done, try the filter shown above. It should have a 3dB point of about 16kHz, and $v_{out}$ should drop like a rock at higher frequencies. Measure its 3dB frequency, then measure its response at $f = 2f_{3dB}$ and at $f = 10f_{3dB}$—if you can; you may not
be able to see any response beyond about $4f_{3\text{dB}}$. Compare these measurements against the rather "soft" response of the $RC$ low-pass filter you measured in section 2.4, and against the calculated response (i.e., ratio of output amplitude to input amplitude) of the two filters shown in the table below.

By far the best way to enjoy the performance of this filter is to sweep the frequency in, as suggested in section 2-4, above. You may, incidentally, find the theoretically-flat "passband" —the frequency range where the filter passes nearly all the signal— not quite flat; a slope or bump here results from our settling for standard values of $R$ and $C$, and 10% capacitors, rather than using exactly the values called for.

---

**Note:** One must give a common-sense redefinition to "$f_{3\text{dB}}$" of this LC filter, because the filter attenuates to $1/2$ amplitude even at DC. One must define the LC's $3$dB point as the frequency at which the output amplitude is down $3$dB *relative to its amplitude at DC*. This is not the last time you will need to modify the meaning of "3dB point" to give it a common sense reading. You will do so again when you meet amplifiers, for example, and discuss their frequency responses.

---

### Rolloff of two filters: 5-pole vs simple $RC$

<table>
<thead>
<tr>
<th>Frequency:</th>
<th>0</th>
<th>$f_{3\text{dB}}$</th>
<th>$2f_{3\text{dB}}$</th>
<th>$4f_{3\text{dB}}$</th>
<th>$10f_{3\text{dB}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$RC$</td>
<td>1.0</td>
<td>0.71</td>
<td>0.45</td>
<td>0.24</td>
<td>0.10</td>
</tr>
<tr>
<td>5-pole</td>
<td>1.0</td>
<td>0.71</td>
<td>0.031</td>
<td>0.001</td>
<td>0.00001</td>
</tr>
</tbody>
</table>

*Figure L2.12: Amplitude out vs amplitude-at-DC: $RC$ filter vs 5-pole LC filter*

---

**Optional: Contrast Ordinary $RC$ against 5-Pole LC Filter**

To appreciate how good the 5-pole filter is, it helps a lot to watch it against an ordinary $RC$ filter. Here is an ordinary $RC$ low-pass filter with $f_{3\text{dB}}$ the same as the Butterworth's: about 16 kHz.

![RC low pass schematic](image)

*Figure L2.13: RC low pass with $f_{3\text{dB}}$ matched to the LC filter's*

Sweep the input frequency, and watch the outputs of the two filters simultaneously on the scope's two channels. This will challenge your scopesmanship: you no longer can use X-Y mode; instead you will need to use the function generator's ramp to trigger the scope.
Class 3: Diode Circuits

Topics:

- old:
  - Once again, the problem of A driving B; this time, A is frequency-dependent

new:

- scope probe; designing it; Fourier helps us check probe compensation
- LC circuit: highly selective; fun to use as "Fourier analyzer"
- Diode circuits

Old:

Remember our claim that our \(10\times\) rule of thumb would let us design circuit fragments? Let’s confirm it by watching what happens to a filter when we violate the rule.

Suppose we have a low-pass filter, designed to give us \(f_{3dB}\) a bit over 1kHz (this is a filter you built last time, you’ll recall):

\[
\begin{align*}
15\,k & \quad 0.01\,\mu F \\
\end{align*}
\]

\[
\begin{align*}
15\,k & \quad 15\,k = 7.5\,k \\
\end{align*}
\]

\[
\begin{align*}
\frac{V_{in}}{2} & \quad 0.01 \\
\end{align*}
\]

Figure N3.1: RC low-pass: not loaded versus loaded; redrawn to simplify

If \(R_{\text{load}}\) is around 150k or more, the load attenuates the signal only slightly, and \(f_{3dB}\) stays put. But what happens if we make \(R_{\text{load}} \) 15k? Attenuation is the lesser of the two bad effects. Look at what happens to \(f_{3dB}\):

\[
\begin{align*}
V_{out}/V_{in} & \quad f_{3dB} - \text{original} \quad f_{3dB} - \text{loaded} \\
\end{align*}
\]

Figure N3.2: Excessive loading shifts \(f_{3dB}\)
**New:**

**A. Scope Probe**

*Text exercise AE-8 (Ch 1)*

Lab 3-8

That mishap leads nicely into the problem of how to design a scope probe.

Until now, we have fed the scope with BNC cables. They work, but at middling to high frequencies they don't work well. Their heavy capacitance (about 30 pF/foot) burdens the circuits you look at, and may make those circuits misbehave in strange ways—oscillating, for example. So, we nearly always use "10×" probes with a scope: that's a probe that makes the scope's input impedance 10× that of the bare scope. (The bare scope looks like 1MΩ parallel about 120 pF—cable and scope.)

Here is a defective design for a 10× probe:

![Figure N3.3: Crummy 10x probe](image)

Do you see what's wrong? It works fine at DC. But try redrawing it as a Thevenin model driving a cap to ground, as in the example we did at the start of these notes. The flaw should appear. What is $f_{3dB}$?

**Remedy**

We need to make sure our probe does not have this low-pass effect: scope and probe should treat alike all frequencies of interest (the upper limit is set by the scope's maximum frequency: for most in our lab that is up to 50 or 60 MHz; a few are good to 100 MHz).

The trick is just to build two voltage dividers in parallel: one resistive, the other capacitive. At the two frequency extremes one or the other dominates (that is, passes most of the current); in between, they share. But if each delivers $V_{in}/10$, nothing complicated happens in this "in-between" range.

![Figure N3.4: Two dividers to deliver $V_{in}/10$ to the scope](image)

What happens if we simply join the outputs of the two dividers? Do we have to analyze the resulting composite circuit as one, fairly messy thing? No. No current flows along the line that joins the two dividers, so things remain utterly simple.
So, a good probe is just these two dividers joined:

![Probe Circuit Diagram]

Figure N3.5: A good ×10 probe: one capacitor is trimmable, to allow use with scopes that differ in $C_{in}$

Practical probes make the probe’s added $C$ adjustable. This adjustment raises a question: how do you know if the probe is properly adjusted, so that it treats all frequencies alike?

**Probe "compensation: Fourier again**

One way to check the frequency response of probe and scope, together, would be to sweep frequencies from DC to the top of the scope’s range, and watch the amplitude the scope showed. But that requires a good function generator, and would be a nuisance to set up each time you wanted to check a probe.

The easier way to do the same task is just to feed scope and probe a *square* wave, and then look to see whether it looks square on the scope screen. If it does, good: all frequencies are treated alike. If it does not look square, just adjust the trimmable $C$ in the probe until the waveform *does* look square.

![Square Wave Diagram]

Figure N3.6: Using square wave to check frequency response of probe and scope

Neat? This is so clearly the efficient way to check probe *compensation* (as the adjustment of the probe’s $C$ is called) that every respectable scope offers a square wave on its front panel. It’s labeled something like *probe comp* or *probe adjust*. It’s a small square wave at around 1kHz.
B. Applying Lab 3’s LC resonant circuit

As the lab notes point out, this circuit is highly selective, passing a narrow range of frequencies (the characteristic called “Q” describes how narrow: \( Q = f_0 / \Delta f_{3\text{dB}} \); \( f_0 \) is the resonant frequency—the favored frequency; \( \Delta f \) is the width at the amplitude that delivers half-power). It’s entertaining to apply the circuit so as to confirm one of Fourier’s claims.

Finding Fourier Components of a Square Wave

This resonant circuit can serve as a “Fourier Analyzer:” the circuit’s response measures the amount of 16 kHz (approx.) present in an input waveform.

Try driving the circuit with a square wave at the resonant frequency; note the amplitude of the (sine wave) response. Now gradually lower the driving frequency until you get another peak response (it should occur at 1/3 the resonant frequency) and check the amplitude (it should be 1/3 the amplitude of the fundamental response). With some care you can verify the amplitude and frequency of the first five or six terms of the Fourier series. Can you think of a way to calculate \( \pi \) with this circuit?

Classier: Frequency Spectrum Display

If you sweep the square wave input to your 16kHz-detector, you get a sort of inverse frequency spectrum: you should see a big bump at \( f_{\text{resonance}} \), a smaller bump at 1/3 \( f_{\text{resonance}} \), and so on.

Our detector-frequency is fixed. The square wave frequency changes. We get one or another of the several frequency components of the square wave. Make sense?
C. Diode Circuits

Diodes do a new and useful trick for us: they allow current to flow in one direction only:

\[ \text{(yes)} \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow 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**Full-wave bridge rectifier**

This clever circuit gives a second bump out, on the *negative* swing of the input voltage. Clever, isn’t it? Once you have seen this output, you can see why the simpler rectifier is called *half-wave*.

![Figure N3.11: Full-wave bridge rectifier](image)

Rarely is there an excuse for using anything other than a full-wave bridge in a power supply, these days. Once upon a time, “diode” meant a $5 vacuum tube, and then designers tried to limit the number of these that they used. Now you just buy the bridge: a little epoxy package with four diodes and four legs; a big one may cost you a dollar. In a minute, we’ll proceed to looking at a full power supply circuit, which will include one of these bridges. But before we do, let’s note one additional sort of diode: the zener.

**Zener diodes**

*Text sec. 2.04¹*

Here is the $I-V$ curve for a zener, a diode that breaks down at some low reverse voltage, and likes to! If you put it into a voltage divider “backwards”—that is, “back-biased:” with the voltage running the wrong way on the one-way street—you can form a circuit whose output voltage is pretty nearly constant, despite variation on the input, and despite variation in loading.

![Figure N3.12: Zener voltage source (Text fig. 2.11)](image)

Here is a chance to use a subtler view of the diode’s behavior. A closer look at the diode’s curve will show us, for example, why we need to follow the text’s rule of thumb that says ‘keep at least 10 mA flowing in the zener, even when the circuit is loaded.’

---

¹ Yes, this is a forward reference; we’d just like you to think at one time about all the diodes that you’ll meet.
The curve turned on its side shows a slope that is $\Delta V/\Delta I$—the device's *dynamic resistance*. That value describes how good a voltage reference the zener is: how much the output voltage will vary as current varies. (Here, current varies because of two effects: variation of *input* voltage, and variation in *output current*, or loading.) Can you see how badly the diode would perform if you wandered up into the region of the curve where $I_{\text{zener}}$ was very small?

We'll show you a worked example of such a zener reference (in another section of these notes)—and then we'll drop the subject for a while. A practical voltage source would never (well, hardly ever!) use a naked zener like the one we just showed you; it would always include a transistor or op-amp circuit after the zener, so as to limit the variation in *output current* (see Chapter 2, sec. 2.04, when you get there). The voltage regulators discussed in Chapter 6 used just such a scheme. At least you should understand those circuits the better for having glimpsed a zener today. Now on to the diode application you will see most often.

*The Most Important Diode Application: Power Supply*

Here is a standard unregulated power supply circuit (we’ll learn later just what “unregulated” means: we can’t understand fully until we meet regulated supplies):

*Text sec. 6.11, fig. 6.17,*

Now we’ll look at a way to choose component values. In these notes, we will do the job incompletely, as if we were just sketching a supply; in a *worked example* you will find a similar case done more thoroughly.
Assume we aim for the following specifications:

- $V_{\text{out}}$: about 12 volts
- Ripple: about 1 volt
- $R_{\text{load}}$: 120Ω

We must choose the following values:

- C size (μF)
- transformer voltage ($V_{\text{RMS}}$)
- fuse rating

We will postpone until the Worked Example two less fundamental tasks (because often a ball-park guess will do): specify:

- transformer current rating
- “bleeder” resistor value

Let's start, doing this in stages. Surely you should begin by drawing the circuit without component values, as we did above.

1. **Transformer Voltage:**

   *Compare Text sec. 6.12*

   This is just $V_{\text{out}}$ plus the voltage lost across the rectifier bridge. The bridge always puts 2 diodes in the path. Specify as $\text{rms}$ voltage: for a sine wave, that means $(1/\sqrt{2})(V_{\text{peak}})$.

   ![Transformer voltage diagram]

   Figure N3.15: Transformer voltage, given $V_{\text{out}}$

   Here, that gives $V_{\text{peak}}$ at the transformer of about 14V ≈ 10V$_{\text{rms}}$

2. **Capacitor:**

   *Compare Text sec. 6.13*

   This is the interesting part. This task could be hard, but we'll make it pretty easy by using two simplifying assumptions.

   Here is what the “ripple” waveform will look like:

   *Text sec. 1.28, fig. 1.73, p. 46*

   ![Ripple diagram]

   Figure N3.16: Ripple
We can figure the ripple (or choose a C for a specified ripple, as in this problem) by using the general equation

\[ I = C \frac{dV}{dt} \]

"dV" or \( \Delta V \) is ripple; "\( dT \)" or \( \Delta T \) is the time during which the cap discharges; "\( I \)" is the current taken out of the supply. (You can, instead, memorize the Text’s ripple formula; but that’s probably a waste of memory space.)

To specify exactly the C that will allow a specified amount of ripple at the stated maximum load requires some thought. Specifically,

- What is \( I_{\text{out}} \)? If the load is resistive, the current out is not constant, but decays exponentially each half-cycle.
- What is \( \Delta T \)? That is, for how long does the cap discharge (before the transformer voltage comes up again to charge it)?

We will, as usual, coolly sidestep these difficulties with some worst-case approximations:

- We assume \( I_{\text{out}} \) is constant, at its maximum value;
- We assume the cap is discharged for the full time between peaks.

Both these approximations tend to overstate the ripple we will get. Since ripple is not a good thing, we don’t mind building a circuit that delivers a bit less ripple than called for.

Try those approximations here:

- \( I_{\text{out}} \) is just \( I_{\text{out-max}}: 100 \text{ mA} \)
- \( \Delta T \) is 1/2 the period of the 60 Hz input waveform: \( 1/120\text{Hz} = 8 \text{ ms} \).

What cap size does this imply?

\[ C = I \Delta T / \Delta V \approx 0.1 \cdot 8 \cdot 10^{-3} / 1 \text{V} \approx 0.8 \cdot 10^{-3} \text{F} = 800 \mu\text{F} \]

That may sound big; it isn’t, for a power supply filter.

3. **Fuse Rating**

The current in the primary is smaller than the current in the secondary, by about the same ratio as the primary voltage is larger. (This occurs because the transformer dissipates little power: \( P_{\text{in}} \approx P_{\text{out}} \).)

\[ P_{\text{IN}} \approx P_{\text{OUT}} \]

\[ \Rightarrow I_{\text{IN}} \cdot V_{\text{IN}} \approx I_{\text{OUT}} \cdot V_{\text{OUT}} \]

**Figure N3.17:** Transformer preserves power, roughly; so, a ‘step down’ transformer draws less current than it puts out.

In the present case, where the voltage is stepped down about 11x, the primary current is lower than the secondary current by about the same factor. So, for 100 mA out, about 9 mA must flow in the primary.
RMS Heating

Text sec. 1.28, 
p. 47; compare exercise 1.28 and figure 1.76

In fact, the fuse feels its "9 mA" as somewhat more, because it comes in surges (see the Text at p.47, and Worked Example below); call it 20 mA. And we don’t want the fuse to blow under the maximum-current load. So, use a fuse that blows at perhaps 4X the maximum $I_{\text{out}}$, adjusted for its heating effect: 80 mA. A 100 mA, or 0.1 A fuse is a pretty standard value, and we’d be content with that. The value is not critical, since the fuse is for emergencies, in which very large currents can be expected.

It’s a good idea to use a slow-blow type, since on power-up a large initial current charges the filter capacitor, and we don’t want the fuse to blow each time you turn on the supply.
Chapter 1: Worked Example: Power Supply

Another Power Supply

Here we will do a problem much like the one we did more sketchily in the Notes for Class 3. If you are comfortable with the design process, skip to sections 5 and 6, below, where we meet some new issues.

We are to design a standard unregulated power supply circuit (we’ll learn later just what “unregulated” means when we meet voltage regulators in Chapter 6). In this example we will look a little more closely than we did in class, at the way to choose component values. Here’s the particular problem:

**Problem: Unregulated power supply**

Design a power supply to convert 110VAC ‘line’ voltage to DC. Aim for the following specifications:

- $V_{out}$: no less than 20 volts
- Ripple: about 2 volts
- $I_{load}$: 1A (maximum)

Choose—

- $C$ size ($\mu$F)
- Transformer voltage ($V_{rms}$)
- Fuse rating ($I$)
- “bleeder” resistor value
- Transformer current rating

**Questions:** What difference would you see in the circuit output—

- If you took the circuit to Europe and plugged it into a wall outlet: there, the line voltage is 220V, 50Hz.
- If one diode in the bridge rectifier burned out (open, not shorted)?

0. **Skeleton Circuit**

First, as usual, we would draw the circuit without component values; you will find that figure in the Class Notes (fig. N3.14). Fuse goes on primary side, to protect against as many mishaps as possible—including failures of the transformer and switch. Always use a full-wave rectifier (a bridge); most of the half-wave rectifier circuits you see in textbooks are relics of the days when “diode” meant an expensive vacuum tube. Now that diode means a little hunk of silicon, and they come as an integrated bridge package, there’s rarely an excuse for anything but a bridge. A “bleeder” resistor is useful in a lab power supply, which might have no load: you want to be able to count on finding close to 0 volts a few seconds after you shut power off. The bleeder achieves that. Many power supplies are always loaded, at least by a regulator and perhaps by the circuit they were built to power; these supplies are sure to discharge their filter capacitors promptly and need no bleeder.
1. Transformer Voltage:

This is just the peak value of $V_{out}$ plus the two diode drops imposed by the bridge rectifier, as usual. If $V_{out}$ is to be 20V after ripple, then $V_{out(\text{peak})}$ should be two volts more: around 22V. The transformer voltage then ought to be about 23V.

When we specify the transformer we need to follow the convention that uses $V_{rms}$, not $V_{peak}$ ($V_{rms}$ defines the DC voltage that would deliver the same power as the particular waveform). For a sine wave, $V_{rms}$ is $V_{peak}/\sqrt{2}$, as you know.

In this case,

$$V_{peak}/\sqrt{2} = 23V/1.4 = 16V_{rms}$$

This happens to be a standard transformer voltage.

If it had not been standard, we would have needed to take the next higher standard value, or use a transformer with a ‘tapped primary’ that allows fine tuning of the step-down ratio.

2. Capacitor:

Here is the “ripple” waveform, again. We have labeled the drawing with reminders that $\Delta t$ depends on circumstances: so, $\Delta t$ varies under the changed conditions suggested in the questions that conclude this exercise.

Using

$$I = C \frac{dv}{dt}$$

we plug in what we know, and solve for $C$. We know—

- “$dV$” or $\Delta V$, the ripple, is 2V;
- “$dt$” or $\Delta T$ is the time between peaks of the input waveform: $1/(2\cdot60Hz) = 8$ ms;
- “$I$” is the peak output current: 1A. (This specification of load current rather than load resistance may seem odd, at the moment. In fact, it is typical. The typical load for an unregulated supply is a regulator—a circuit that holds its output voltage constant; if the regulated supply drives a constant resistance, then it puts out a constant current despite the input ripple, and it thus draws a constant current from the filter capacitor at the same time).

Putting these numbers together, we get:

$$C = I \cdot \Delta t / \Delta v = 1A \cdot 8 \cdot 10^{-3} \text{s} / 2V = 4000 \mu\text{F}$$

Big, but not unreasonably big.

3. Fuse Rating

This supply steps the voltage down from 110V to 16V; the current steps up proportionately: about 7. So, the output (secondary) current of 1A implies an input (primary) current of about 140mA.

But this calculation of average input current understates the heating effect of the primary current, and of the primary and secondary currents in the transformer. Because these
currents come in surges, recharging the filter capacitor during only a part of the full cycle, the currents during the surges are large. These surges heat a fuse more than a steady current delivering the same power, so we need to boost the fuse rating by perhaps a factor of 2 to 4, and then another factor of about 2 to prevent fuse from blowing at full load. (It's designed for emergencies.)

This set of rules of thumb carry us to something like——

\[
\text{fuse rating (current)} = 140\text{mA} \cdot 4\text{(for current surges)} \cdot 2\text{(not to blow under normal full load)} \\
\approx 1.1\text{A}
\]

A 1A slow-blow would do. Why slow-blow? Because on power-up (when the supply first is turned on) the filter capacitor is charged rapidly in a few cycles; large currents then flow. A fuse designed to blow during a brief overload would blow every time the supply was turned on. The slow-blow has larger thermal mass: needs overcurrent for a longer time than the normal fuse, before it will blow.

4. **Bleeder Resistor**

Polite power supplies include such a resistor, or some other fixed load, so as not to surprise their users. Again the value is not critical. Use an R that discharges the filter cap in no more than a few seconds; don't use a tiny R that substantially loads the supply.

Here, let's let \( RC = \text{a few seconds} \). \( \Rightarrow R = \text{a few}/C \approx 1\text{k} \)

Before we go on to consider a couple of new issues, let's just draw the circuit with the values we have chosen, so far:

![Power supply: the usual circuit, with part values inserted](image)

5. **Transformer Current Rating**

This is harder. The transformer provides brief surges of current into the cap. These heat the transformer more than a continuous flow of smaller current. Here is a sketch of current waveforms in relation to two possible ripple levels:

![Transformer Current versus Ripple: Small Ripple ==> Brief High-Current Pulses, and excessive heating](image)
The left hand figure show current flowing for about 1/5 period, in pulses of 5A, to replace the charge drained at the steady 1 A output rate. The right hand figure shows current flowing for 1/20 period, in pulses of 20 A.

*Moral:* a little ripple is a good thing. You will see that this is so when you meet voltage regulators, which can reduce ripple by a factor between 1000 and 10,000. To say this another way, a volt of ripple out of the unregulated supply may look like less than a millivolt at the point where it goes to work (where the output of the regulated supply drives some load)!

6. Questions: What difference would you see in the circuit output—

1. If you took it to Europe, where the line voltage is 220V, 50Hz?
2. If one diode in the bridge rectifier burned out (open, not shorted)?

Solutions:

1. In Europe, the obvious effect would be a doubling of output voltage. That’s likely to cook something driven by this supply (that’s why American travelers often carry small 2:1 step-down transformers). It might also cook transformer and filter capacitor, unless you had been very conservative in your design (it would be foolish, in fact, to specify a filter cap that could take double the anticipated voltage; caps grow substantially with the voltage they can tolerate).

So you probably would not have a chance to get interested in less obvious changes in the power supply. But let’s look at them anyway: the output ripple would change: \( \Delta T \) would be 1/50Hz = 10ms, not 8ms.

Ripple should grow proportionately. If load current remained constant, ripple should grow to about 2.5V. If the load were resistive, then load current would double with the output voltage, and ripple would double relative to the value just estimated: to around 5V.

2. The burned out diode would make the bridge behave like a *half-wave* rectifier. \( \Delta t \) would double, so ripple amplitude would double, roughly. Ripple frequency would fall from 120Hz to 60Hz. (This information might someday tell you what’s wrong with an old radio: if it begins to buzz at you at 60Hz, perhaps half of the bridge has failed; if it buzzes at 120Hz, probably the filter cap has failed. If you like such electronic detective work, many pleasures lie ahead of you.)
Lab 3: Diode Circuits

Reading: Finish Chapter 1, including “Power in reactive circuits” (pp 33-34)
Appendix E

Problems: Problems in text.
Additional Exercises 7,8.

3-1. LC Resonant Circuit.

![LC Circuit Diagram]

Figure L3.1: LC parallel resonant circuit

Construct the parallel resonant circuit shown above. Drive it with a sine wave, varying the frequency through a range that includes what you calculate to be the circuit’s resonant frequency. Compare the resonant frequency that you observe with the one you calculated. (The circuit attenuates the signal considerably, even at its resonant frequency; the L is not perfectly efficient, but instead includes some series resistance.)

Use the function-generator’s sweep feature to show you a scope display of amplitude-out versus frequency. (See Lab 2 notes, if you need some advice on how to do this trick.)

When you succeed in getting such a display of frequency response, try to explain why the display grows funny wiggles on one side of resonance as you increase the sweep rate. Clue: the funny wiggles appear on the side after the circuit has already been driven into resonant oscillation; the function generator there is driving an oscillating circuit.

Finding Fourier Components of a Square Wave

This resonant circuit can serve as a “Fourier Analyzer:” the circuit’s response measures the amount of 16 kHz (approx.) present in an input waveform.

Try driving the circuit with a square wave at the resonant frequency; note the amplitude of the (sine wave) response. Now gradually lower the driving frequency until you get another peak response (it should occur at 1/3 the resonant frequency) and check the amplitude (it should be 1/3 the amplitude of the fundamental response). With some care you can verify the amplitude and frequency of the first five or six terms of the Fourier series. Can you think of a way to calculate pi with this circuit?
Here is a reminder of the Fourier series for a square wave:

\[ \frac{4}{\pi} \sin \omega t + \frac{4}{3\pi} \sin 3\omega t + \frac{4}{5\pi} \sin 5\omega t + \cdots \]

Figure L3.2: Fourier series for square wave

**Classier: Frequency Spectrum Display**

If you sweep the square wave input to your 16kHz-detector, you get a sort of inverse frequency spectrum: you should see a big bump at \( f_{\text{resonance}} \), a smaller bump at \( 1/3 f_{\text{resonance}} \), and so on.

"Ringing"

Now try driving the circuit with a low-frequency square wave: try 20 Hz. You should see a brief output in response to each edge of the input square wave. If you look closely at this output, you can see that it is a decaying sine wave. (If you find the display dim, increase the square wave frequency to around 100 Hz.)

What is the frequency of this sine wave? (No surprise, here.)

Why does it decay? Does it appear to decay exponentially?

You will see such a response of an LC circuit to a step input whenever you happen to look at a square wave with an improperly grounded scope probe: when you fail to ground the probe close to the point you are probing, you force a ground current to flow through a long (inductive) path. Stray inductance and capacitance form a resonant circuit that produces ugly ringing. You might look for this effect now, if you are curious; or you might just wait for the day (almost sure to come) when you run into this effect inadvertently.

### 3-2. Half-wave Rectifier.

![Half-wave rectifier diagram](image)

Construct a half-wave rectifier circuit with a 6.3Vac (rms) transformer and a 1N914 diode, as in the figure above. Connect a 2.2k load, and look at the output on the scope. Is it what you expect? Polarity? Why is \( V_{\text{peak}} > 6.3V \)? (Don’t be troubled if \( V_{\text{peak}} \) is a bit more than 6.3V\( \cdot \sqrt{2} \): the transformer designers want to make sure your power supply gets at least what’s advertised, even under heavy load; you’re loading it very lightly.)
3-3. Full-wave Bridge Rectifier.

Now construct a full-wave bridge circuit, as above. Be careful about polarities—the band on the diode indicates cathode, as in the figure. Look at the output waveform (but don’t attempt to look at the input—the signal across the transformer’s secondary—with the scope’s other channel at the same time; this would require connecting the second “ground” lead of the scope to one side of the secondary. What disaster would that cause?). Does it make sense? Why is the peak amplitude less than in the last circuit? How much should it be? What would happen if you were to reverse any one of the four diodes? (Don’t try it!).

Don’t be too gravely alarmed if you find yourself burning out diodes in this experiment. When a diode fails, does it usually fail open or closed? Do you see why diodes in this circuit usually fail in pairs—in a touching sort of suicide pact?

Look at the region of the output waveform that is near zero volts. Why are there flat regions? Measure their duration, and explain.

3-4. Ripple.

Now connect a 15μF filter capacitor across the output (Important—observe polarity). Does the output make sense? Calculate what the “ripple” amplitude should be, then measure it. Does it agree? (If not, have you assumed the wrong discharge time, by a factor of 2?)

Now put a 500μF capacitor across the output (again, be careful about polarity), and see if the ripple is reduced to the value you predict. This circuit is now a respectable voltage source, for loads of low current. To make a “power supply” of higher current capability, you’d use heftier diodes (e.g., 1N4002) and a larger capacitor. (In practice you would always follow the power supply with an active regulator, a circuit you will meet in Lab 12.)

3-5. Signal Diodes.

Use a diode to make a rectified differentiator, as in the figure above. Drive it with a square wave at 10kHz or so, at the function generator’s maximum output amplitude. Look at input and output, using both scope channels. Does it make sense? What does the 2.2k load resistor do? Try removing it.

Hint: You should see what appear to be RC discharge curves in both cases—with and without the 2.2k to ground. The challenge here is to figure out what determines the R and C that you are watching.
3-6. Diode Clamp.

![Diode Clamp Circuit](image)

Construct the simple diode clamp circuit shown just above. Drive it with a sine wave from your function generator, at maximum output amplitude, and observe the output. If you can see that the clamped voltage is not quite flat, then you can see the effect of the diode’s non-zero impedance. Perhaps you can estimate a value for this dynamic resistance (see Text sec. 1.); try a triangle waveform, if you attempt this estimate.

![Clamp with Voltage Divider Reference](image)

Now try using a voltage divider as the clamping voltage, as shown just above. Drive the circuit with a large sine wave, and examine the peak of the output waveform. Why is it rounded so much? (Hint: What is the impedance of the “voltage source” provided by the voltage divider? If you are puzzled, try drawing a Thevenin model for the whole circuit. Incidentally, this circuit is probably best analyzed in the time domain.) To check your explanation, drive the circuit with a triangle wave; compare with figure 1.83 in the text.

As a remedy, try adding a 15μF capacitor, as shown with dotted lines (note polarity). Try it out. Explain to your satisfaction why it works. (Here, you might use either a time- or frequency-domain argument.) This case illustrates well the concept of a bypass capacitor. What is it bypassing, and why?

3-7. Diode Limiter.

![Diode Limiter Circuit](image)

Build the simple diode limiter shown above. Drive it with sines, triangles, and square waves of various amplitudes. Describe what it does, and why. Can you think of a use for it?
3-8. Impedances of Test Instruments.

We mentioned in the first lab that measuring instruments (voltmeters, ammeters) ideally should leave the measured circuit unaffected. For instance, this implies an infinite impedance for voltmeters, and zero impedance for ammeters. Likewise, an oscilloscope should present an infinite input impedance, while power supplies and function generators should be zero-impedance sources.

Begin by measuring the internal resistance of the VOM on its 10V dc range. You won't need anything more than a dc voltage and a resistor, if you're clever.

Next try the same measurement on the 50V dc range. Make sense? (Most needle-type VOM's are marked with a phrase such as "20,000 ohms per volt" on their dc voltage ranges; remember this, from the first class?). For further enlightenment, see the Box on Multimeters (text pp. 9-10).

Now use a similar trick to measure the input resistance of the scope. Remember that it should be pretty large, if the scope is a good voltage measuring-instrument. As a voltage source use a 100Hz sine wave, rather than a dc voltage as above.

To measure the scope's input impedance drive it with a signal in series with 1 megohm (figure 3.9, above). What is the low frequency (f < 1kHz) attenuation? Now raise the frequency. What happens? Explain, in terms of a model of the scope input as an R in parallel with a C. What are the approximate values of R and C? What remedy will make this circuit work as a divide-by-two signal attenuator at all frequencies? Try it!

Now go back and read the section entitled "Probes" in the Text's Appendix A. Then get a 10X probe, and use it to look at the calibrator signal (usually a 1V, 1kHz square wave) available on the scope's front panel somewhere. Adjust the probe "compensation" screw to obtain a good square wave. Use 10X probes on your scope in all remaining lab exercises, like a professional!

Finally, measure the internal resistance of the function generator. Don't try to do it with an ohmmeter! Instead, load the generator with a known resistor and watch the output drop. One value of $R_{load}$ is enough to determine $R_{internal}$, but try several to see if you get a consistent value. Use a small signal, say 1 volt pp at 1kHz.
Ch. 1 Review: Important Topics

Important Topics

1. Resistive Circuits:
   a. voltage dividers
   b. $R_{\text{out}}, Z_{\text{out}}, R_{\text{in}}, Z_{\text{in}}$: Thevenin models

2. RC Circuits
   a. Generally
      i. time-domain vs frequency domain
      ii. step-response vs sine response
   b. Important RC Circuits
      i. integrator, differentiator: (either can be described as a filter that is murdering signal)
      ii. filters
         1. $f_{3\text{dB}}$
         2. phase-shift
         3. phasors (only an optional aid to visualizing what’s going on)

3. Diode Circuits
   a. rectifiers
   b. clamp
   c. zener voltage reference
   d. power-supply
      i. ripple
      ii. transformer rating: rms
      iii. current ratings: fuse, transformer

4. LC Circuits
   Rare in this course: one LC resonant circuit (Lab 3)
   But they haunt us as unwanted effect of stray L and C: ringing in circuits;
   oscillation in a follower, e.g. (see ‘unwanted oscillations’ in Lab 10)
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<th>Ch.1: Jargon and terms</th>
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