Billiards: A 2-dimensional mechanical analog to Rutherford scattering

Some lower division undergraduate physics labs include a 2-D “billiards” experiment which is analogous to the Rutherford scattering experiment you will be performing in physics 122. Of course some of the details are different! This document is from one such lower division lab.

Particle Scattering

Table of Contents

Nobel Prize:

Lord Earnest Rutherford was awarded the Nobel Prize in 1908 for his "investigations in regard to the decay of elements and the chemistry of radioactive substances."

He was a talented, hard-working physicist with enormous drive and self-confidence. In a letter written later in life, he wrote, "I've been reading some of my early papers and, you know, when I'd finished, I said to myself, "Rutherford, my boy, you used to be a damned clever fellow." Though pleased at winning a Nobel Prize he was not happy that it was a chemistry prize, rather than one in physics. In his speech accepting the prize he noted that he had observed many transformations in his work with radioactivity but never had seen one as rapid as his own, from physicist to chemist.

HARD-SPHERE MODEL OF ATOMIC SCATTERING

LABORATORY EXPERIMENT

Introduction

This apparatus is designed to acquaint the user with some of the mechanics of scattering of atomic particles by using analogous objects of large dimensions. The diameter of a target is determined from data supplied by the scatter pattern of projectiles deflected by it.

Scattering experiments analogous to this simple mechanical one are most useful in many areas of physics. Much knowledge of nuclei, electrons, protons, neutrons and alpha particles and observing what occurs at different angles. By so doing, definite quantitative conclusions about the scatters are obtained although they cannot actually be seen. Of course a single particle is never fired at a particular scatterer using a predetermined impact parameter, as is done in this experiment, but rather a large number of particles bombard the target and the desired information is obtained from the relative number scattered at a given angle.

Theory

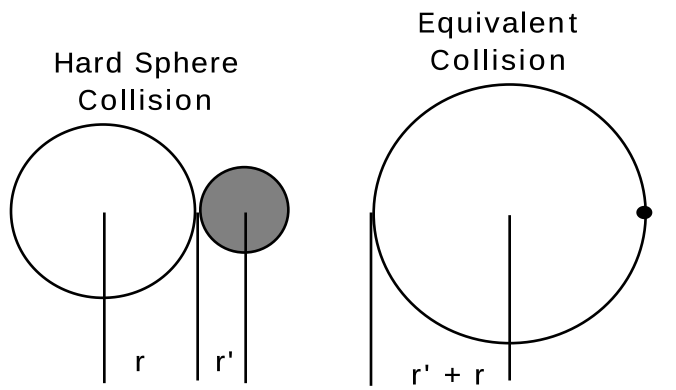
Theory of Hard Sphere Collisions

The energy transfer between colliding hard spheres can take place by either elastic or inelastic scattering. In a collision where there is energy transfer but the kinetic energy is conserved, this collision is classified as being perfectly **elastic**. In an elastic collision particles transfer kinetic energy determined by the laws of conservation of momentum and kinetic energy.
If the colliding particles transfer energy to some other form of energy than kinetic energy then this leaves the bombarding particle and target particle with less energy to share. This interaction is called **inelastic**, and the kinetic energy for the colliding particles is not conserved, however, the conservation of momentum is conserved in the inelastic collision. The collisions between many particles in a system can be considered to occur at random. The probability that one particle suffers a collision with some other particle during any small time interval is thus assumed to be independent of its history of past collisions. The mean time *tau (*τ*)* which the particle travels before suffering its next collision is called the *mean free time* of the particle. The mean distance λ which the particle travels before suffering its next collision is called the *mean free path* of the particle. If the particle has some mean speed υ, then the mean free path λ and the mean free time τ are related by

$$\lambda = \upsilon\tau$$

where the velocity of the particle is measured relative to the laboratory frame of reference. This is not the relative velocity between the particles and is not measured relative to the atoms. The mean time between collisions tau τ will be determined using the relative velocity between the bombarding particles and the target particles.

It will be assumed for hard sphere scattering that the two particles do not interact through any long range force, but that the forces of interaction are only those between two hard spheres of radii r' and r. A collision between two particles is considered to take place whenever one particle makes contact with the other. Two rigid spheres will collide if their centers pass within a distance "d" (r' + r) of each other. It is this center-to-center distance that determines a collision, therefore, it is possible to replace an actual collision with the equivalent collision geometry as shown in the following figure, in which the incoming particle has been reduced to a point and the target particle is expanded to a sphere of radius d = r' + r.



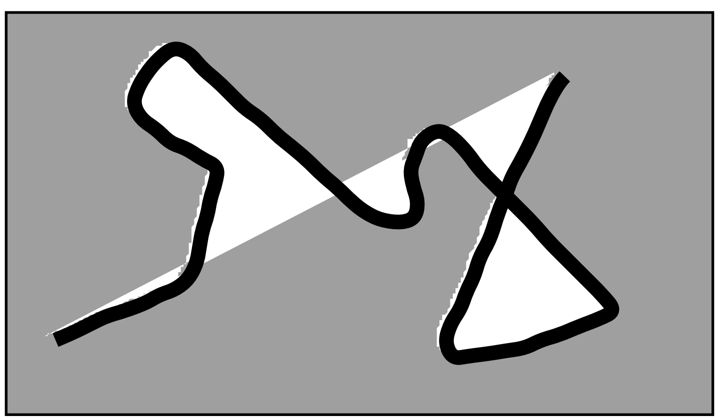
A bombarding particle with and equivalent diameter "d" which travels a long distance L will sweep out a volume

$$V = \pi d^2 L$$

where the equivalent collision area πd^2 is called the collision cross sectional area sigma (σ). The scattering cross section will collide with any other particle whose center lies within the volume. If n is the concentration of target particles, the average number of particles in this volume is

$$N = n\sigma L$$

and this is the number of collisions as illustrated in the following figure:



The volume swept out by the collision cross section

The average distance between collisions, the mean free path, is the total path length traveled divided by the total number of collisions

$$N = n\sigma L$$

and the mean free path becomes

$$\lambda = \frac{1}{n\sigma} \quad where \quad \sigma = \pi d^2 .$$

Therefore the mean free path is small when the number of target particles per unit volume is large, there are more particles for which the bombarding particle can collide, and the mean free path is also small when the diameter of the target particle is large, the bombarding particle is more likely to interact with a target particle.

In the above derivation for the mean free path of the bombarding particle it was assumed that the target particles are at rest relative to the incoming particles. This is not true in general, since both the bombarding particle and the target particles move, their relative mean speed V is different from the mean speed v of an individual particle. If these velocities are taken into consideration the mean free path for the bombarding particles becomes:

$$\lambda = \left(\frac{\bar{v}}{\bar{V}_{rel}}\right)\frac{1}{n\sigma}$$

where the relative velocity of between the bombarding particle and the target particle is given by

$$\vec{V} = \vec{v} - \vec{v}' .$$

The above result is obtained from the following derivation. The definition of the mean free path of the electrons is:

$$\lambda = \bar{v}\tau$$

where v bar is the mean speed of the electrons relative to the tube. This is not measured relative to the atoms. The mean time between collisions tau will take care of the velocities of the atoms as follows.

The two particle collision interaction can be replaced by the equivalent scattering cross section sigma (σ). As this scattering cross section moves through out the volume of target particles it will sweep out a volume. All target particles within this volume will have a collision. This scattering cross section will move at a velocity that must be measured relative to the target particles and is given by

$$\vec{V} = \vec{v} - \vec{v}'$$

where v is the velocity of the bombarding particle and v' is the velocity of the target particles.

The volume swept out by the scattering cross section σin a time "t" is

$$Vol = \sigma L$$

where L is given by $L = V_{rel}t$ and $V_{rel} = \left|\vec{V}\right| = \left|\vec{v} - \vec{v}'\right|$

so the relative velocity is $V_{rel} = \sqrt{\left(v^2 - 2\vec{v} \cdot \vec{v}' + v'^2\right)}$.

Therefore the number of particle collisions that takes place in the volume with "n" target particles per unit volume is

$$\#col = n \ \ Vol = n \ \ \sigma L = n \ \ \sigma V_{rel} t \, .$$

If the time "t" that the sweeping cross section moves through the target particles is equal to the mean free time τ then the number of collisions is limited to only ONE by definition that the mean free time is the average time between collisions, and for this special condition, there can only be one collision so the following special relationship is true by definition:

$$1 = n \ \ \sigma \ \ V_{rel} \ \ \tau$$

The expression for the mean free time τ can be deduced from the above special relationship:

$$\tau = \frac{1}{n\sigma V_{rel}} \, .$$

Substituting this value for the mean free time between collisions into the expression for the mean free path, the following relationship is determined:

$$\lambda = \left(\frac{\overline{v}}{\overline{V}_{rel}}\right)\frac{1}{n\sigma}$$

where v is the mean velocity of the target particles and V_{rel} is the mean relative velocity of the bombarding particles with respect to the target particles. The relative velocity is given by

$$V_{rel} = \sqrt{\left(v^2 - 2\vec{v} \cdot \vec{v}' + v'^2\right)}$$

and the mean of the relative velocity squared is

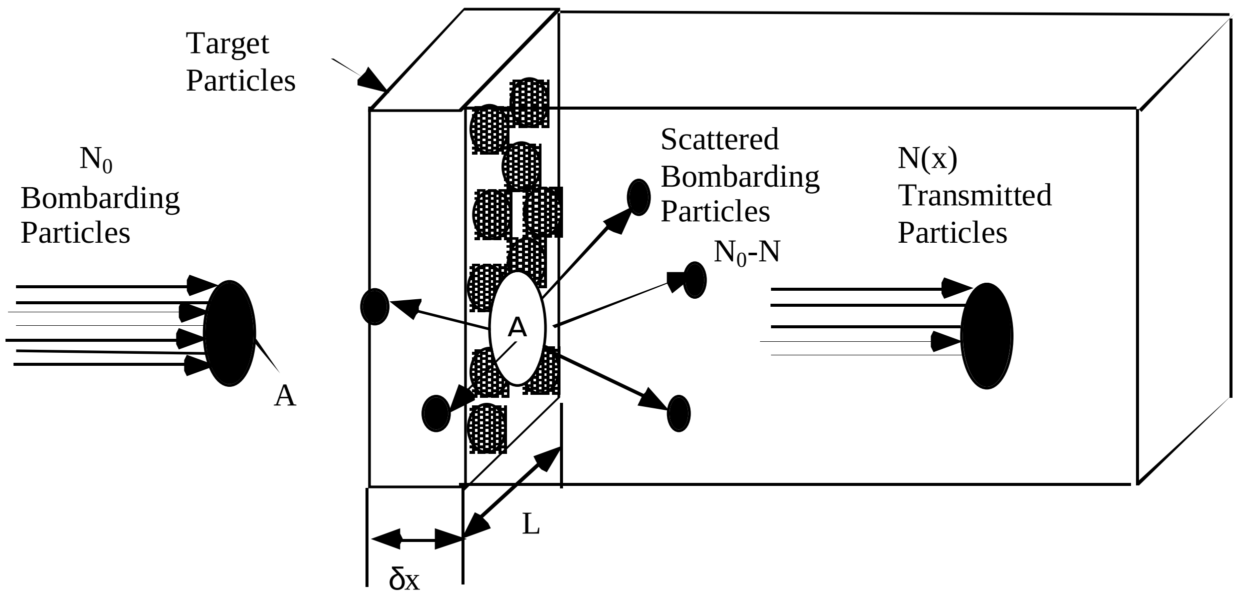
$$\overline{V_{rel}^2} = \overline{v^2} + \overline{v'^2} - \overline{2\vec{v} \cdot \vec{v}'}$$

and the last term $\overline{\vec{v} \cdot \vec{v}'} = 0$, since the cosine of the angle between v and v' is as likely not be positive as negative.

A more analytical method of analyzing the collision process is as follows. A beam of bombarding particles of total number N, really an electron flux, is incident upon a volume of space filled with target particles., a gas of "n" target particles per unit volume. Assume that the bombarding particles will interact via *elastic* and *inelastic* collisions with the target particles through the hard sphere model as proposed above. Most of the bombarding particles will pass through the target particle gas, but some will have elastic and inelastic collisions and these collisions will remove some of bombarding particles δN from the incoming particle beam.

Consider a thin layer of target particles of area A by δx. The ratio of the number of collisions, δN, to the total number of incident particles, N, is equal to the ratio of the total scattering cross section area to the covered area of the thin layer volume:

$$\frac{\delta N}{N} = \frac{ScatteringCrossSectionArea}{TargetArea}$$



The equivalent scattering cross section area of one target particle is:

$$\sigma = \pi(r' + r)^2$$

The total collision cross section is the product of this and the number of target particles in the thin layer:

$$\# = nA\delta x$$

The total area of the thin target layer covered by the incident particles is also just A, so

$$\frac{\delta N}{N} = n\sigma\delta x$$

where δN / N is the fraction number of bombarding particles that have collisions with the target particles and are removed from the electron beam, therefore, this ratio does represent the probability of a collision.

If N_0 bombarding particles per unit area are incident normally on the face of a layer of material containing target particles at rest with a total scattering cross section "nσ", then the number N of transmitted bombarding particles per unit area through a finite thickness "x" will be given by

$$\int_{N_0}^{N} -\left(\frac{\delta N}{N}\right) = \int_0^x n\sigma\delta x\,,$$

integrating to give the number of bombarding particles along the scattering distance x:

$$N = N_0 e^{-n\sigma x}$$

which results in an exponential decrease in bombarding particles with distance into the target volume.

The number of bombarding particles that collide with target particles is δN in the distance δx, therefore, the total ranges of the bombarding particles in this group is just δN*δx. The average distance traveled by the bombarding particles will be the sum of the combined ranges of all groups from N_0 to 0 divided by the total number of bombarding particles, which is

$$\bar{x} = \lambda = \frac{\int_{N_0}^{0} x\,dN}{\int_{N_0}^{0} dN}$$

The number of bombarding particles as a function of "x" is given above

$$N = N_0 e^{-n\sigma x}$$

and the derivative dN is given by

$$dN = -N_0 n\sigma e^{-n\sigma x} dx\,.$$

The integral for the mean free path becomes:

$$\lambda = n\sigma \int_0^{\infty} x e^{-n\sigma x} dx$$

which can be integrated to obtain the expression for the mean free path

$$\lambda = \frac{1}{n\sigma}$$

where n is the number of target particles per unit volume and σ is the equivalent scattering cross section for hard sphere scattering.

Theory for the Experiment

In this experiment steel balls are shot at a plastic (Lucite) target in a plane perpendicular to the axis of the target. Each ball is deflected by the target and strikes a paper strip attached to the chamber wall, making an indentation, the position of which enables one to determine the "scattering angle" _ through which the ball has been deflected (see Figure 1).

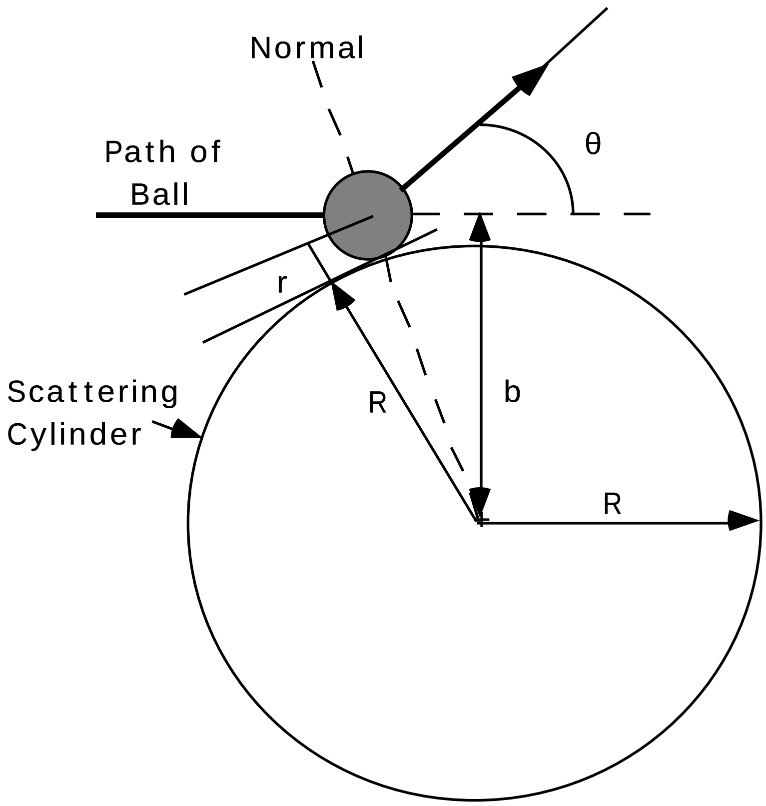


Figure 1

From measurements of the angle θ, the impact parameter b, and the radius r of the steel ball, together with certain assumptions about the nature of the scattering process, one can determine R the radius of the target. Thus the size of the target may be measured indirectly by means of the angular distribution of the ball scattered from it. Assume that when the ball strikes the target the impact is elastic and that a reflection process is operative. Making use of these assumptions, an equation describing the scattering of the steel balls may be derived. Since a reflection process has been assumed, angle $ß_1 = ß_2$. Also since it can be shown that triangle ODC is congruent to triangle OBC, angle α_1 = angle α_2 and therefore arc DM = arc MB as shown in Figure 2.

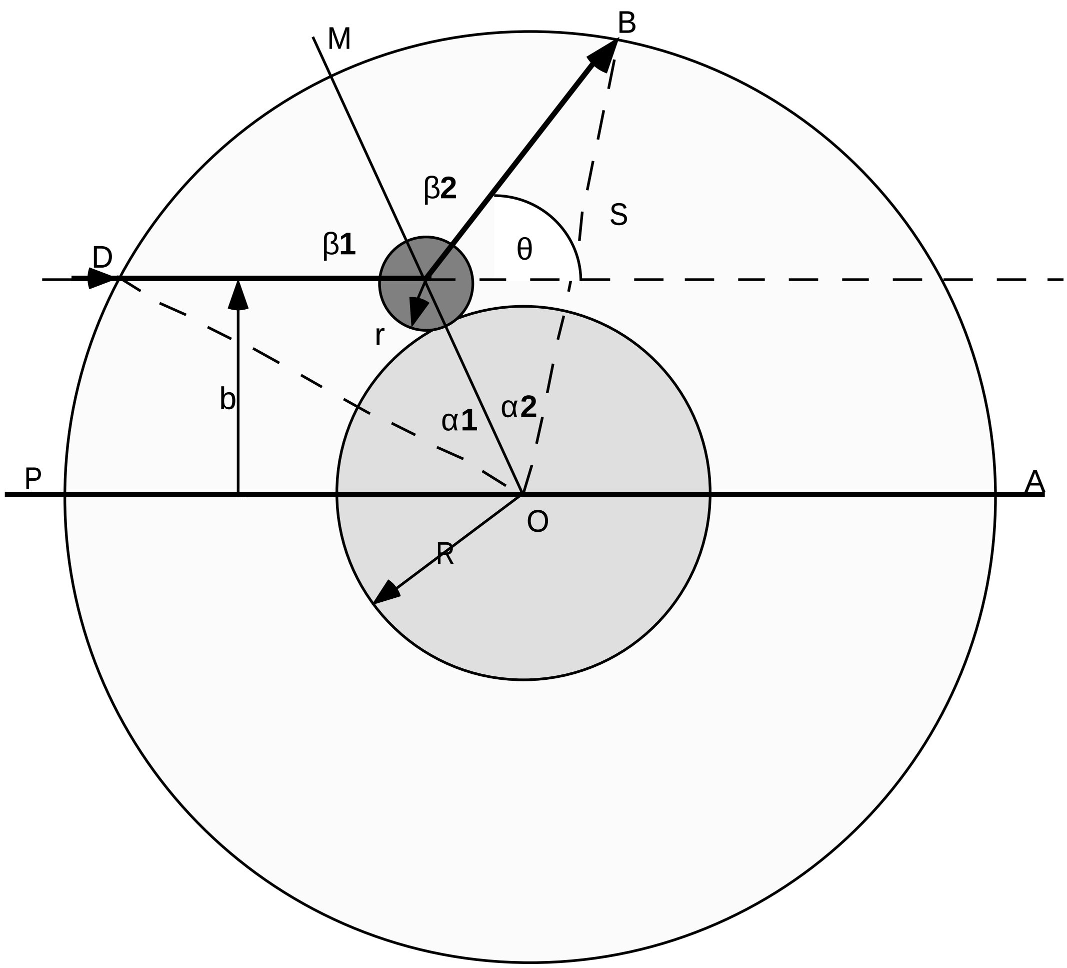


Figure 2

When using the target with the largest diameter the impact parameter b may vary to an maximum for which the angle POD is approximately 5^o. For an angle of this size the sine of the angle equals the angle and b/S = arc PD/S or b = arc PD. Since 5^o is the largest value of the angle POD, the above is true for all targets, up to 2-1/2 inches in diameter.

From Figure 2 the scattering angle θ is

$$\theta = \pi - 2\beta_1 \; or \; \frac{\theta}{2} = \frac{\pi}{2} - \beta_1 \qquad (1)$$

_ $$\cos\left(\frac{\theta}{2}\right) = \sin\beta_1 \qquad (2)$$

However $$\sin\beta_1 = \frac{b}{R + r} \qquad (3)$$

and $$\cos\left(\frac{\theta}{2}\right) = \frac{b}{R+r} \qquad (4)$$

In this experiment θ cannot be measured directly but $ß_1$ can be determined in terms of the arc AB, the impact parameter, and the radius of the chamber wall.

Again from Figure 2 we have

$$ArcPM = \frac{\pi S - arcAB - b}{2} + b$$

$$ArcPM = \frac{\pi S - arcAB + b}{2}$$

$$and \quad angle \;\; \beta_1 = \frac{arcPM}{S} = \frac{\pi S - arcAB + b}{2s}$$

These equations make it possible to calculate the scattering angle θ and the radius R of the target in terms of measurable quantities.

Apparatus

The apparatus consists of a cylindrical chamber (large flat plastic tub) mounted on a non-wrapping base. A horizontal slot in the chamber wall provides an opening for the bulb-operated air gun mounted on the base outside the chamber. The gun may be moved laterally by means of the screw adjustment. The gun is used to project 0.181-inch steel balls at the cylindrical target in the center of the chamber. The target is polished Lucite, this material being better for this purpose than hardened steel.

A roll of 2 inch wide paper tape, a roll of double-sided adhesive tape, a supply of 0.181-inch steel balls, and two 5/32-inch diameter alignment rods are included.

Procedure

SETTING UP THE APPARATUS. The scattering apparatus is shipped assembled and essentially ready for use. The axis of the gun is made perpendicular to the axis of the screw in assembly. It is necessary, however, to establish the zero position of gun and tape with reference to the center of the cylinder. By means of the double-sided adhesive tape attach a strip of wax-coated tape along the lower edge of the chamber wall on the side opposite the gun. Place the longer rod in the groove in the target and place the other in the barrel of the gun making one end almost touch the center post. Move the gun until one rod is approximately over the other. Rotate the target, if necessary, until the rods are exactly parallel. Now make final adjustment of the gun position to make the rods lie in the same vertical plane. The gun in this position is aimed directly at the center of the target and the impact parameter b is zero. The far end of the rod in the groove will locate the zero position of the tape from which angles in either direction are measured.

The value of the impact parameter b that is a measure of the distance the gun is displaced from its zero position, is determined by the pitch of the screw and the number of turns. The screw has 18 threads per inch which, after converting to the metric system, means that **the gun moves 0.141 cm per revolution of screw.**

To load the gun rotate the metal sleeve until the hole in the sleeve lines up with the hole in the barrel, drop in the ball and rotate the sleeve to close the hold. The apparatus should be so leveled that the barrel of the gun is horizontal; otherwise the ball may roll from the barrel before it can be fired. Best results are obtained when the hose of the aspirator is held straight and the balls are not fired too hard.

1. Attach a fresh strip of tape to the chamber wall using a length sufficient to cover about three-fourths of the circumference, with waxed side out. Locate and mark zero position on the tape. Move the gun to the position for b = 0. In this position the ball should collide head-on with the target and be deflected back to strike the wall directly beneath the gun.

2. The purpose of the experiment is to obtain data to check the validity of equation (4) and determine the value of R, the radius of the target. Fire the gun for a series of values of the impact parameter, firing several shots for each value, enabling cylinder, that is, use impact parameters on both sides of the zero position. Be sure to correlate the impact parameter with the grouping of shots obtained for that value. After each shot locate the dent made in the paper by the ball and label it so that you can identify it later.

3. Remove the tape and measure the distance from the zero position on the tape to each indentation. Average these values for each group. Record these as values of the arc AB against corresponding values of the impact parameter b.

4. Using equation (6) calculate values of the angles $ß_1$ for corresponding values of the arc AB. Now using equation (1) calculate corresponding values of $\theta/2$. **Plot cos $\theta/2$ against b and determine R + r**. Compare this value with that obtained by direct measurement.

The extent to which equation (4) describes our data provides us with an indication of the validity of our assumptions.

HARD-SPHERE MODEL OF ATOMIC CATTERING

DIFFERENTIAL CROSS SECTION IN 2-D

LABORATORY EXPERIMENT

Introduction

In the previous experiment the scattering apparatus was used to determine the diameter of the target from the scatter pattern of projectiles deflected from the target and the known impact parameter. In the real physical scattering experiment it is impossible to know the impact parameter of the incoming particle. Research in nuclear physics measure the number of scattered particles at a known angle and from this data they can calculate the differential cross section of the scattering nucleus. In this experiment, the exact conditions of an experiment in nuclear physics will be simulated as close as possible. The impact parameter will not be measured as this is not possible in any real atomic scattering experiment.

Theory

Differential Cross Section

In the real nuclear scattering experiment it is impossible to know the impact parameter of the incoming particles. A particle detector is placed opposite the scattering material at known angles and the number of scattered projectile particles are detected. This detector has an angular width $\Delta\theta$, so it actually counts the particles that enter through the conical wedge bounded by θ and $\theta + \Delta\theta$. These particles have come from the beams whose impact parameters lie between b and $b - \Delta b$ as shown in Figure 1.

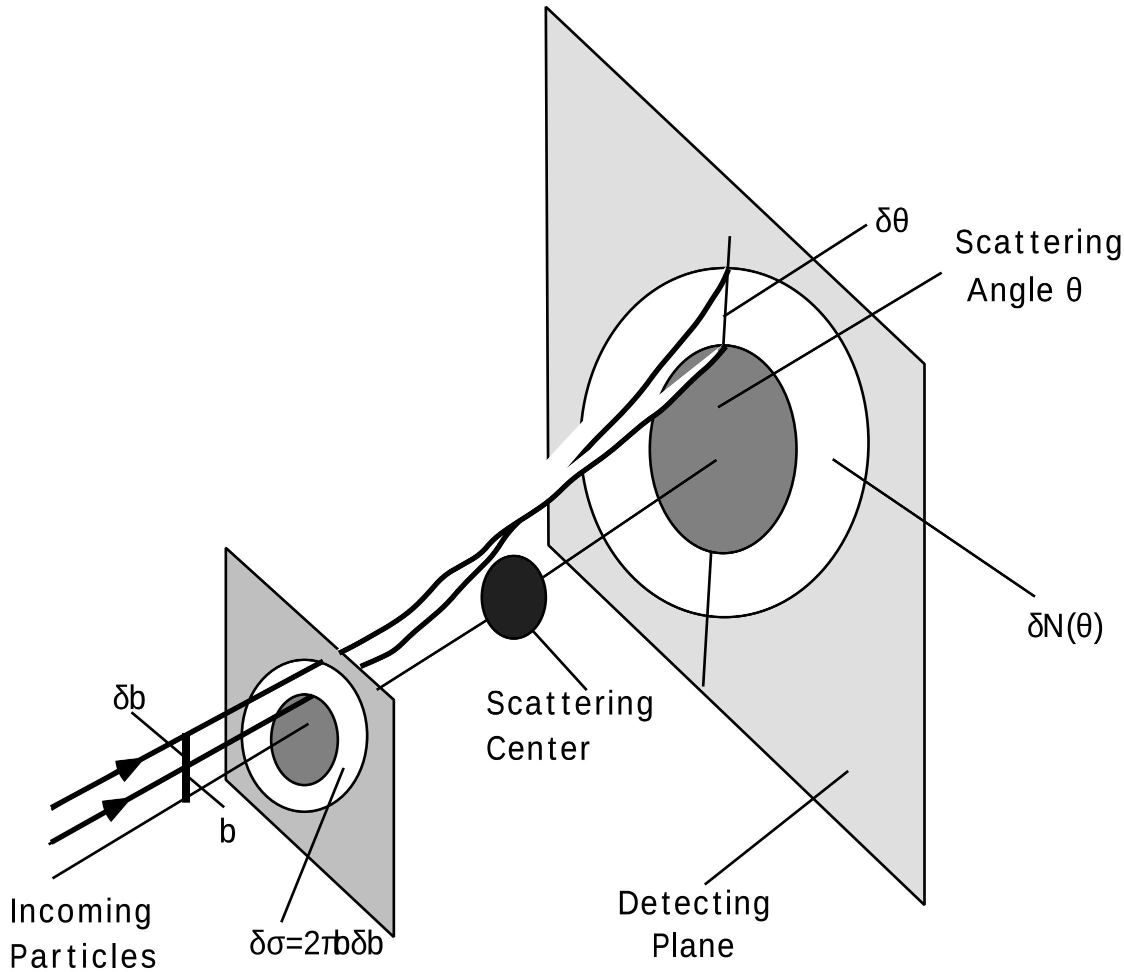


Figure 1

The washer-like area $\Delta\sigma = 2\pi b \cdot \Delta b$ is called the scattering cross-section, and is related to ΔN, the number of particles scattered into the angle θ and θ + Δθ, by

$$\Delta N(\theta) = I \cdot \Delta\sigma = I \cdot 2\pi b \cdot \Delta b \qquad (1)$$

where I is the particle flux, the number of particles per unit area normal to the beam per second.

The differential scattering cross-section is

$$\frac{\Delta\sigma}{\Delta\Omega} = \frac{1}{I}\frac{\Delta N(\theta)}{\Delta\Omega} = \frac{b}{\sin\theta}\cdot\frac{\Delta b}{\Delta\theta} \qquad (2)$$

where ΔΩ is the solid angle, defined as the

$$(\text{area} / r^2) = 2\pi\sin\theta\Delta\theta. \qquad (3)$$

Differential Cross Section in 2-D

The experimental apparatus used to simulate the scattering experiment is not an exact reproduction of the physical nuclear experiment. This experiment will be carried out in only two dimensions. Therefore, the equation for the cross-section must be modified for two-dimensional scattering; the "unit area" must be replaced by "unit distance". To simulate an isotropic beam of incoming particles, several ball bearings are fired at a large number of different impact parameters. The strength of the beam is thus measured in units of shots per cm, and the "cross-section" in units of cm. The incoming particles will be in a vertical line, and will not cover a cross sectional area. The incoming particle flux I will be determined from the total number of particles shot N over a total impact distance B;

$$I = \frac{N}{B} \left(\frac{\text{particles}}{\text{cm}}\right) . \qquad (4)$$

Therefore, the number of particles scattered in Δb through θ and $\theta + \Delta\theta$ and will be counted $\Delta N(\theta)$ is

$$\Delta N(\theta) = I \cdot \Delta b = \left(\frac{N}{B}\right) \cdot \Delta b \quad (\#\text{counted in } \Delta\theta \text{ at } \theta) \qquad (5)$$

Since all of the particles incident upon the length Δb are scattered through the angle $\Delta\theta$, the 2-D scattering cross-section is simply Δb;

$$\Delta\sigma \equiv \Delta b \qquad (6)$$

and since the one dimensional detector at θ has an angular width of $\Delta\theta$, the 2-D differential scattering cross-section becomes

$$\frac{\Delta\sigma}{\Delta\theta} = \frac{\Delta b}{\Delta\theta} . \quad \left(\begin{array}{l}\text{2-D differential}\\ \text{cross-section}\end{array}\right) \qquad (7)$$

The experimentally determined differential cross-section for 2-D scattering is determined from the detected particles $\Delta N(\theta)$ in $\Delta\theta$ divided by the linear flux,

$$\left(\frac{\Delta\sigma}{\Delta\theta}\right)_{\text{exp}} = \frac{1}{I}\frac{\Delta N(\theta)}{\Delta\theta} = \left(\frac{B}{N}\right)\frac{\Delta N(\theta)}{\Delta\theta} \qquad (8)$$

The 2-D differential scattering cross-section can also be expressed in a functional relationship between the parameters for the scattering geometry. This analytical form must be determined from the physics and the geometry of the particle interaction between the incoming projectile and the scattering center. The 2-D differential cross-section was derived above as the change in the impact parameter (Δb) with respect to the scattering angle ($\Delta\theta$). The differential cross-section for a specific interaction between the projectile and the target can be determined when a specific relationship is developed between the impact parameter b and the scattering angle θ.

In the 2-D hard sphere collision laboratory experiment, we will assume that the impact is elastic and that the angle of incidence (α) is equal to the angle of reflection (α). The hard sphere collision details are shown in Figure 2.

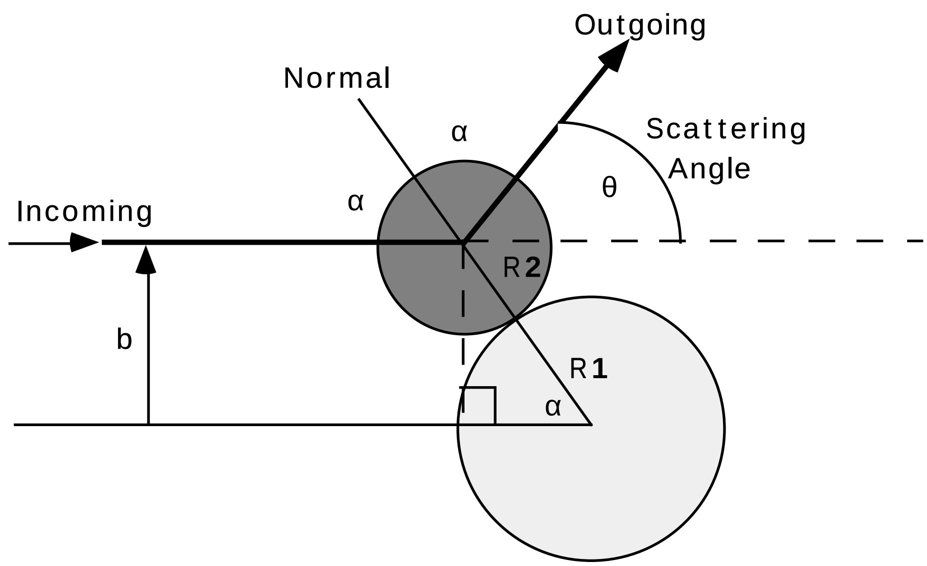


Figure 2

From the scattering geometry as shown in Figure 2, the scattering angle θ and the angle of incidence are related by

$$2\alpha=\pi-\theta \tag{9}$$

Using the right angle triangle formed by the radii of the two spheres at the point of closest approach, the incident angle α is related to the impact parameter b by

$$\sin(\alpha) = \frac{b}{R_1 + R_2} \tag{10}$$

The functional relationship between the impact parameter b and the scattering angle θ is obtained by combining these two expressions,

$$b = (R_1 + R_2)\ \ Sin\left(\frac{\pi}{2} - \frac{\theta}{2}\right)\ , \tag{11}$$

or

$$b = (R_1 + R_2)\ \cos\left(\frac{\theta}{2}\right) . \tag{12}$$

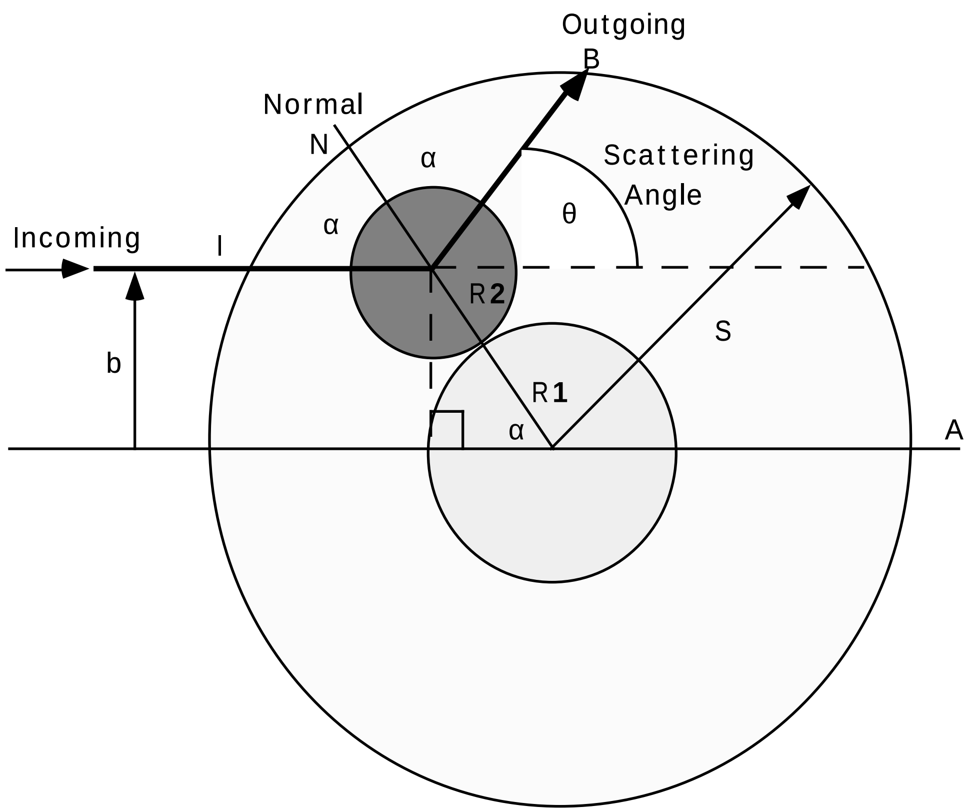
The 2-D differential cross-section is the absolute value of the change in b with respect to θ,

$$\frac{\Delta\sigma}{\Delta\theta} = \left|\frac{\Delta b}{\Delta\theta}\right| = \frac{1}{2}(R_1 + R_2) \cdot \sin\left(\frac{\theta}{2}\right) \qquad (13)$$

The <u>2-D differential cross-section is expressed in a theoretical form; Equation (13) and in a experimental form; Equation (8)</u>. A comparison of the values obtained from Equations (8) and (13) furnishes a means of checking experiment against theory.

<u>Scattering Angle</u>

The evaluation of the scattering angle from measurements made around the perimeter of the detection chamber must be considered in more detail. Figure 3 shows the geometry of the detection chamber. The particles are incident at point I and strike the target at the center of the chamber and are then scattered to the point B on the perimeter. As shown in Figure 3, the incident particle angle is related to the arc IN when measured from the center of the scattering chamber.



<u>Figure 3</u>

If we take two times the arc IN and add the arc AB, this will be equal to one-half the circumference plus an extra impact distance b. (b ≈ arc b)

$$2 \cdot \text{arc IN} + \text{arc AB} = \pi S + b \qquad (14)$$

Solving for the arc IN

$$\text{arc IN} = \frac{1}{2}(\pi S - arcAB + b) \qquad (15)$$

The incident particle angle α is related to the arc IN by

$$\alpha = \frac{\text{arc IN}}{\text{S}} \qquad (16)$$

The scattered particle angle θ is also related to the incident particle angle α,

$$\theta = \pi - 2\alpha \qquad (17)$$

Measuring the arc AB will determine the scattering angle,

$$\theta = \pi - 2\alpha = \pi - 2\frac{arcIN}{S}$$
$$\theta = \pi - \frac{\pi S - arcAB + b}{S} \qquad (18)$$
$$\theta = \frac{arcAB}{S} - \frac{b}{S}$$

In a real atomic scattering experiment the radius of the detecting chamber S is many times greater than the collision impact parameter b. The scattering angle is very easily calculated from the arc distance along the perimeter of the chamber to the detection location.

Experimental Apparatus

The apparatus consists of a cylindrical chamber wall with a Lucite target mounted in the center. A horizontal slot in the chamber wall provides an opening for a air gun. The gun may be moved laterally by means of a screw -0.141 cm per revolution. The gun projects a 0.181 inch steel ball at the cylindrical target. Special marking tape is used as the detector of the scattered projectile. The steel ball will leave a mark on the tape at the point of collision around the circumference of the chamber.

Procedure

1. Set up - It is necessary to establish the zero position of gun and tape with reference to the center of the scattering chamber. Place a small piece of detection take along the chamber wall on the side opposite the gun. Place the longer steel rod in the groove in the target and place the other in the barrel of the gun making one end almost touch the center post. Move the gun and target until the rods lie in the same vertical plane. The far end of the long rod in the groove will locate the zero position of the tape from which the scattering angles can be measured. The best results are obtained when the projectiles are not fired too hard.

2. Attach a fresh strip of marking tape around the entire circumference of the chamber with the white side out. Locate and mark the zero position on the tape. Measure the radius of the scattering chamber (S).

3. To simulate an isotropic beam of incoming particles, several ball bearings are fired at a large number of different impact parameters. The impact parameters are chosen so that the width of the beam is greater than the scatterer and to ensure isotropy, the same number of ball must be fired from each position of the sun and the gun is traversed the same distance before each firing. Starting with the gun in the zero position, fire about four balls at each position with one-quarter turn of the gun-traversing screw between positions.

4. Remove the tape and determine the number of scattered particles $\Delta N(\theta)$ within $\Delta\theta$ at θ. Divide the circumference into equal increments of 10 or 20 degrees and count the number of marks within these increments.

5. The differential cross section is determined by counting the number of marks as a function or linear distance along the paper. The scattered angle is determined using the method discussed in the theory section.

6. Make a histogram plot of the number of scattered particles verses the angle of scatter. Compare to the theoretical prediction that the number of scattered particles should show a $\sin(\theta/2)$ relationship.

7. Determine the total "cross-section" both experimentally and theoretically and compare the results.