Optical Pumping

MIT Department of Physics
(Dated: August 6, 2004)

Measurement of the Zeeman splittings of the ground state of the natural rubidium isotopes; measurement of the relaxation time of the magnetization of rubidium vapor; and measurement of the local geomagnetic field by the rubidium magnetometer. Rubidium vapor in a weak (~0.01-10 gauss) magnetic field controlled with Helmholtz coils is pumped with circularly polarized D1 light from a rubidium rf discharge lamp. The degree of magnetization of the vapor is inferred from a differential measurement of its opacity to the pumping radiation. In the first part of the experiment the energy separation between the magnetic substates of the ground-state hyperfine levels is determined as a function of the magnetic field from measurements of the frequencies of rf photons that cause depolarization and consequent greater opacity of the vapor. The magnetic moments of the ground states of the $^{85}$Rb and $^{87}$Rb isotopes are derived from the data and compared with the vector model for addition of electronic and nuclear angular momenta. In the second part of the experiment the direction of magnetization is alternated between nearly parallel and nearly antiparallel to the optic axis, and the effects of the speed of reversal on the amplitude of the opacity signal are observed and compared with a computer model. The time constant of the pumping action is measured as a function of the intensity of the pumping light, and the results are compared with a theory of competing rate processes - pumping versus collisional depolarization.

1. PREPARATORY PROBLEMS

1. With reference to Figure 1 of this guide, estimate the energy differences (order of magnitude) in eV between each of the following pairs of states of Rb:

   (a) $5^2S_{1/2} - 5^2P_{3/2}$
   (b) $5^2P_{1/2} - 5^2P_{3/2}$
   (c) $5^2S_{1/2}(f = 2) - 5^2S_{1/2}(f = 3)$
   (d) $5^2S_{1/2}(f = 2, m_f = -1) - 5^2S_{1/2}(f = 2, m_f = 0)$ in a magnetic field of 1 gauss.

2. Derive the Landé g-factors for the ground state of the two rubidium isotopes using either the vector model or matrix mechanics for the addition of angular and magnetic moments.

3. In thermal equilibrium at 320 K how many atoms in a mole of rubidium would one expect to find in the $5^2P_{1/2}$ state? What is the difference in the population of the lowest and highest magnetic substate of the ground state in a field of 1 gauss at that temperature?

4. Given two plane polarizing filters and a quarter wave plate, how do you make a beam of circularly polarized light?

5. What fractional contributions does the nucleus of a rubidium atom make to its (a) angular momentum (b) magnetic moment?

6. If the earth’s field were that of a magnetic dipole at the center and parallel to the rotation axis, then what would be the direction of the field at the latitude of Boston?

2. INTRODUCTION

A. Kastler discovered optical pumping in the 1950’s and received the Nobel Prize for his work in 1966. In this experiment you will explore the phenomenon of optical pumping and its application to fundamental measurements in atomic and nuclear physics with equipment originally built for the Junior Lab shortly after the publication of Kastler’s work. Various improvements have been made since then. Though its components are simple, the apparatus is capable of yielding accurate results that illustrate several of the fundamental principles of quantum theory and atomic structure. Among other possibilities it can be used to measure collisional relaxation phenomena, and the magnitude and direction of the local geomagnetic field.

3. ANGULAR MOMENTUM AND ATOMIC STRUCTURE

We will be concerned with the angular momentum and energy of neutral atoms of rubidium in a vapor under conditions in which each atom can be treated as a nearly isolated system. Such a system in free space has eigenstates with definite values of the square of the total angular momentum, $\vec{F} \cdot \vec{F}$, and of the component of angular momentum with respect to some given direction, $F_z$. According to the laws of quantum mechanics the eigenvalues of $\vec{F} \cdot \vec{F}$, and $F_z$ are given by the expressions

$$\vec{F} \cdot \vec{F} = f(f + 1)\hbar^2$$  \hspace{1cm} (1)

and

$$F_z = m_f \hbar$$  \hspace{1cm} (2)
where \( f \) is an integer or half-integer, and \( m_f \) has one of the \( 2f+1 \) values \(-f, -f+1, \ldots, f-1, f\). In the absence of any magnetic field, the eigenstates with the same \( f \) but different \( m_f \) are degenerate, i.e., they have the same energy. However, an atom with angular momentum \( F \) generally has a magnetic moment

\[
\vec{\mu} = g_f \frac{e}{2m} \vec{F}
\]

where \( m \) is the electron mass, \( e \) is the magnitude of the electron charge, and \( g \) is a proportionality constant called the Landé \( g \) factor. If an external magnetic field is present, then the degeneracy will be lifted by virtue of the interaction between the external magnetic field and the magnetic moment of the atom. If the external field is sufficiently weak so that it does not significantly alter the internal structure of the atom, then the energy of a particular magnetic substate will depend linearly on the field strength, \( B \), and the value of \( m_f \) according to the equation

\[
E = E_0 + g_f m_f \mu_B B
\]

where \( E_0 \) is the energy of the eigenstate in zero field and \( \mu_B \equiv \frac{e\hbar}{2m} \approx 9.1 \times 10^{-21} \text{ergs/gauss} \) is the Bohr magneton. The \( g \) factor depends on the quantum numbers of the particular state and is of order unity. The difference in energy between magnetic substates in a magnetic field is called the magnetic, or Zeeman, splitting. By measuring this splitting in a known field one can determine the value of \( g_f \mu_B \). Alternatively, given the value of \( g_f \mu_B \) one can use a measurement of the splitting to determine the magnitude of an unknown magnetic field.

### 3.1. Interactions at the atomic and nuclear scales

A rubidium atom contains nearly 300 quarks and leptons, each with an intrinsic angular momentum of \( \frac{\hbar}{2} \). Triplets of quarks are bound by the “color” force to form nucleons (protons and neutrons) with dimensions of the order of \( 10^{-13} \text{cm} \). The nucleons are bound in nuclei with dimensions of the order of \( 10^{-12} \text{cm} \), and the remnants of the color force which “leak” out of the nucleons. Charged leptons (electrons) and nuclei are bound by electromagnetic force in atomic structures with dimensions of the order of \( 10^{-8} \text{cm} \). Atoms are bound in molecules by electromagnetic force. Each of these structures - nucleons, nuclei, atoms, and molecules - can exist in a variety of states. However, the lowest excited states of nucleons and nuclei have energies so far above the ground states that they are never excited in atomic physics experiments. Moreover, the size of the nucleus is tiny compared to the size of the electronic structure of the atom. Thus, for the purposes of understanding all but the most refined measurements of atomic structure, the nucleus and all it contains can be considered as a point particle characterized by its net charge, angular momentum, magnetic dipole moment, and in some circumstances, weak higher electric and magnetic moments. The problems of atomic physics are thereby reduced to that of understanding how the electrons and nucleus of an atom interact with one another and with external fields.

According to the shell model, the electronic configuration of rubidium is

\[1s^22s^22p^63s^23p^63d^{10}4s^24p^65s = 5^2S_{1/2}\]

which specifies 36 electrons in closed (i.e. subtotal angular momentum=0) shells with principal quantum number \( n \) from 1 to 4 plus a single “optical” electron in a 5s orbital. The letters \( s \), \( p \), and \( d \) denote the electron “orbitals” with orbital angular momentum quantum numbers \( l = 0, 1, \) and 2, respectively. (Note the 5s shell begins to fill before the 4d). The overall designation of the electronic ground state is \( 5^2S_{1/2} \), which is called “five doublet S one-half.” “Five” specifies the principal quantum number of the outer electron. “S” specifies that the total orbital angular momentum quantum number of the electrons is zero. The term “one-half” refers to the subscript 1/2 which specifies the total angular momentum quantum number of the electrons. With only one electron outside the closed shells the total angular momentum of the ground state is due to just the spin of the outer electron. “Doublet” refers to the superscript \( 2=2s+1 \) where \( s=1/2 \) is the total spin angular momentum quantum number; the word characterizes the total spin by specifying the number of substates that arise from the combinations of \( \vec{L} \) and \( \vec{S} \). In the alkali metals this amounts to a two-fold “fine structure” splitting of the energy levels of excited electronic states with \( l \geq 1 \) due to the interaction of the magnetic moments associated with the spin and orbital angular momenta of the outer electron. In such states the orbital and spin angular momenta can combine in just two ways to give \( j = l \pm 1/2 \). Just as in the case of two bar magnets pivoted close to one another, the state of lower energy is the one in which the spin and orbital magnetic moments are anti-parallel and the total (electronic) angular momentum quantum number is \( j = l - 1/2 \). The word “doublet” is retained for S states even though there is no spin-orbit interaction because \( l = 0 \), and consequently the ground state has no fine structure substates \([1]\).

With the shell model as a framework one can explain quantitatively many of the low-energy atomic phenomena, like those involved in the present experiment, in terms of the following hierarchy of interactions, arranged in order of decreasing energy:

1. Interaction between the outer electron(s) and the combined coulomb field of the nucleus and inner closed-shell electrons, giving rise to states specified by the principal quantum numbers and the orbital angular momentum quantum numbers of the outer electrons and differing in energy by \( \Delta E \approx 1 \text{eV} \)

2. Interaction between the magnetic moments associ-
ated with the spin and orbital angular momenta of the outer electron(s), giving rise to “fine structure” substates differing in energy by $\Delta E \approx 10^{-3} eV$.

3. Interaction between the total electronic magnetic moment and the magnetic moment of the nucleus, giving rise to “hyperfine structure” substates with $\Delta E \approx 10^{-6} eV$.

4. Interaction between the total magnetic moment of the atom and the external magnetic field.

The torque resulting from the interaction between the magnetic moments associated with the spin and orbital angular momenta of the electrons causes $\mathbf{L}$ and $\mathbf{S}$ to precess rapidly about their sum $\mathbf{J} = \mathbf{L} + \mathbf{S}$. The much weaker interaction between the magnetic moments associated with $\mathbf{J}$ and the nuclear angular momentum $\mathbf{I}$ causes $\mathbf{J}$ and $\mathbf{I}$ to precess much more slowly about their sum $\mathbf{F} = \mathbf{J} + \mathbf{I}$. And finally a weak external magnetic field $\mathbf{B}$ exerts a torque on the magnetic moment associated with $\mathbf{F}$ that causes it to precess about $\mathbf{B}$ even more slowly. However, if the external field is sufficiently strong that the precession frequency of $\mathbf{F}$ about $\mathbf{B}$ approaches that of $\mathbf{J}$ and $\mathbf{I}$ about $\mathbf{F}$, then the dependencies of the separations between the substates on the external field become nonlinear, and the separations become unequal.

The angular momentum quantum numbers of the rubidium nuclei are $i=5/2$ for $^{85}$Rb and $i=3/2$ for $^{87}$Rb. According to the rules for adding angular momenta, the total angular momentum quantum number of the atom must be one of the values $j + i$, $j + i - 1$, $j + i - 2$,.......$|j - i|$. Thus each electronic state with a certain value of $j$ is split into a number of hyperfine substates equal to $2j+1$ or $2i+1$, whichever is smaller, with a separation in energy between substates that depends on the strength of the magnetic interaction between the total magnetic moment of the electrons and the magnetic moment of the nucleus. Since $j = 1/2$ in the ground state, the number of hyperfine substates is just 2, and the values of $f$ are 5/2+1/2=3 and 5/2-1/2=2 for $^{85}$Rb and 3/2+1/2=2 and 3/2-1/2=1 for $^{87}$Rb.

The magnetic moments of nuclei are of the order of the nuclear magneton ($\mu_N \approx 4\pi m_e c = 5.0 \times 10^{-24}$ ergs/gauss) which is smaller than the Bohr magneton by a factor equal to the ratio of the proton mass to the electron mass, i.e about 2000 times smaller. The magnitude of the magnetic moment of a nucleus is $g_n \mu_n \sqrt{I(I + 1)}$, where $I$ is the angular momentum quantum number of the nucleus, $\mu_n$ is the nuclear magneton, and $g_n$ is a gyromagnetic ratio arising from the configuration of the nucleons themselves. Empirically determined values for $g_n$ are:

$g_n = 1.3527$ for $^{85}$Rb with $I = 5/2$ $g_n = 2.7505$ for $^{87}$Rb with $I = 3/2$.

Thus, regardless of how the electronic and nuclear angular momenta are combined, the nuclear magnetic moment makes a small contribution to the total magnetic moment of the atom. However, the nuclear angular momentum, quantized in units of $h$ like the electronic angular momentum, makes a major contribution to the total angular momentum of the atom and, therefore, to any properties that depend on how the angular momenta are combined such as the Landé $g$ factor and the multiplicities of magnetic substates.

Figures 1 and 2 are schematic energy level diagrams for the ground and lowest excited states of $^{85}$Rb and $^{87}$Rb in a weak magnetic field, showing how the ground and first excited electronic states can be imagined to split into successive sublevels as the various interactions mentioned above are “turned on”. The energy scale is grossly distorted in order to display the hierarchy of structure in one figure. In fact, the separations between the Zeeman levels in a weak external field of $\approx 1$ gauss are $\approx 10^8$ times smaller than the separation between the unperturbed 5S and 5P levels. The right most portions of Figures 1 and 2 represent the specific level structures with which we will be dealing in this experiment.

In the two lowest excited electronic states, designated $5^2P_{1/2}$ and $5^2P_{3/2}$, the valence electron has orbital angular momentum quantum number $l = 1$. This combines with the electron spin to give a total electronic angular momentum quantum number $j$ of 1/2 or 3/2. The energies of these two states differ by virtue of the “fine structure” interaction between the spin and orbital magnetic moments of the outer electron. Transitions from the hyperfine and magnetic substates of $5^2P_{3/2}$ and $5^2P_{1/2}$ to the hyperfine and magnetic substates of 5S produce photons in two narrow optical spectral regions called the rubidium D-lines at 7800 Â and 7948 Å. Transitions between the various energy levels can occur under a variety of circumstances, which include the following of particular interest in this experiment:

1. Electric dipole transitions between the 5S and 5P states; a) either absorption or stimulated emission induced by interactions with optical frequency photons having energies close to the energy difference
between two atomic states. b) spontaneous emission Electric dipole transitions are restricted by the selection rules $\Delta f = \pm 1$, $\Delta m_f = 0$ or $\pm 1$ except not $f = 0$ to $f = 0$, $\Delta m_f = 0$ or $\pm 1$

2. Magnetic dipole transitions between magnetic substates induced by radio frequency photons with energies close to the energy differences of two Zeeman levels The selection rule is $\Delta m_f = \pm 1$.

3. Collision-induced transitions between the magnetic substates.

4. OPTICAL PUMPING

Under conditions of thermal equilibrium at temperature $T$ the distribution of atoms among states of various energies obeys the Boltzmann distribution law according to which the ratio $\frac{n_2}{n_1}$ of the numbers of atoms in two states of energy $E_1$ and $E_2$ is

$$\frac{n_1}{n_2} = e^{\exp \left( \frac{E_2 - E_1}{kT} \right)} \tag{5}$$

where $k = 8.62 \times 10^{-5}$ eV K$^{-1}$ is the Boltzmann constant. With this formula we can calculate the fraction of rubidium atoms in the first excited electronic state in a vapor at room temperature for which $kT \approx 0.03eV$. The first excited state lies about 2 eV above the ground state. The Boltzmann factor is therefore $\exp^{-67} \approx 10^{-29}$, which implies that only about one atom in 10 kilograms is in an excited state at any given moment. On the other hand, the differences between the energies of magnetic substates in the weak fields used in this experiment are very small compared to $kT$ at the temperature of the rubidium vapor. Thus, under equilibrium conditions, there is only a slight difference in the populations of the magnetic substates.

Optical pumping is a process in which absorption of light produces a population of the energy levels different from the Boltzmann distribution. In this experiment you will irradiate rubidium atoms in a magnetic field with circularly polarized photons in a narrow range of energies such that they can induce $5^2S_{\frac{1}{2}} \rightarrow 5^2P_{\frac{1}{2}}$ dipole transitions. However, absorption can occur only if the total angular momentum of the incident photon and atom is conserved in the process. If the incident photons have angular momentum $+\hbar$, the only allowed transitions are those in which $\Delta m_f = 1$. Thus every absorption produces an excited atom with one unit more of projected angular momentum than it had before the transition. On the other hand, the spontaneous and rapid ($\approx 10^{-8}s$) $5^2P_{\frac{1}{2}} \rightarrow 5^2S_{\frac{1}{2}}$ decay transitions occur with only the restriction $\Delta m_f = 0$ or $\pm 1$. The net result is a “pumping” of the atoms in the 5S magnetic substates toward positive values of $m$.

The process is illustrated schematically in Figure 4 which depicts the histories of several atoms which are initially in various magnetic substates of a lower electronic state. Under irradiation by circularly polarized light they make upward transitions to magnetic substates of an upper electronic state subject to the restriction $\Delta m_f = +1$. Spontaneous downward transitions occur with $\Delta m_f = \pm 1,0$. When an atom finally lands in the $m_f = +1$ substate of the lower electronic state it is stuck because it cannot absorb another circularly polarized photon.

The rate of spontaneous transitions among the magnetic substates is small. To reduce the rate of depolarizing collisions, the rubidium vapor is mixed with a “buffer” gas (neon) which has no magnetic substates in its ground electronic state and is therefore unable to absorb or donate the small quanta of energy required for magnetic substate transitions. The buffer atoms shield the rubidium atoms from colliding with one another and slow their diffusion to the walls. With the field in the direction of the beam, the most positive substate may be the one of highest energy. Alternatively, with the field in the opposite direction, that same substate would be the one of lowest energy. In either case the population of the extreme substate, whichever it may be, will increase at the expense of the other substates until, as in the actual experiment, there are few atoms left which can absorb the incident circularly polarized photons. Thus the intensity
of the transmitted beam increases as the absorbing ability of the vapor diminishes, and it approaches an asymptotic value determined by the rate at which depolarizing collisions restore the atoms to substates which can absorb the polarized photons. In effect, the vapor is magnetically polarized and optically “bleached” by the process of optical pumping, and the resulting distribution of the atoms among their possible quantum states is grossly different from the Boltzmann distribution. (The absorption of the photons also gives rise to a non-Boltzmann population of the $5^2 P_{1/2}$ states, but the decay by electric dipole transitions is so rapid that the fraction of atoms in those states at any given time remains extremely small).

In principle, the population of the magnetic substates could be changed by simply turning the beam on and off with a mechanical chopper. Immediately after each turnoff depolarizing collisions would restore thermal equilibrium. When the beam is turned on again one would observe an opacity pulse, i.e. a transmission through the vapor which starts low and increases toward an asymptotic value as the pumping action proceeds and the balance between pumping and collisional depolarization is approached. However, this method has problems caused by the large changes in the signal levels when the pumping beam is turned on and off.

In the present experiment the intensity of the circularly polarized beam is kept steady and changes in the polarization of the vapor are produced by either 1) flooding the vapor with radio photons of the requisite resonant frequency and polarization to induce transitions between the magnetic substates by absorption and/or stimulated emission, or 2) suddenly changing the magnetic field.

With radio photons of the resonance frequency, rapid transitions are induced between the magnetic substates so their populations are kept nearly equal in spite of the pumping action and the vapor can absorb the circularly polarized optical photons. The resonant frequency, indicated by an increase in opacity, is a direct measure of the energy difference between the magnetic substates. Knowing the magnetic field and the resonant frequency, one can derive the value of the magnetic moment of the rubidium atom.

To see how the transitions are induced by a resonance rf field we represent the weak ($\ll 0.2$ gauss) linearly polarized rf magnetic field vector as a sum of two circularly polarized component vectors according to the identity

$$\vec{B}_{rf} = B_{rf}[\cos \omega t, 0, 0] = \frac{B_{rf}}{2}[\cos \omega t, \sin \omega t, 0] + \frac{B_{rf}}{2}[\cos \omega t, -\sin \omega t, 0]$$

where $\omega = \gamma B_0$ is the the Larmor precession frequency of the dipoles in the strong ($B_0 \approx 0.2$ gauss) DC field, $\gamma = \frac{g \nu c}{2 \mu_0 H}$ is the gyromagnetic ratio of the atoms, and $g_F$ is the Landé $g$ factor. If we confine our attention to the circular component of the rf field rotating in the same sense as the Larmor precession of the moments, then we have the situation discussed by Melissinos (1966, page 344) in the context of nuclear magnetic resonance.

The effects of changing the Z-component of the field on the opacity of the vapor undergoing magnetic pumping depend on the speed, sign and amplitude of the change. One can study the behavior of polarized moments in changing fields and the depolarizing interactions in the vapor by observing the size and shapes of the resulting opacity pulses. The effect on the polarization of changing the field can be described as the effect of a changing torque on a gyroscope. It turns out that the motion of the net angular momentum, $\vec{L}$, of an ensemble of atoms in a magnetic field is governed by the classical equation

$$\frac{d\vec{L}}{dt} = \gamma \vec{L} \times \vec{B}$$

where $\vec{B}$ is the magnetic field, and $\gamma$ is the gyromagnetic ratio of the individual atoms (Abragam 1961). If $\vec{B}$ is steady, the motion is a precession about the field direction with angular velocity $\gamma \vec{B}$ and a constant value of the angle between $\vec{L}$ and $\vec{B}$. If $\vec{B}$ changes direction at a rate that is small compared to $\gamma B$, then the angle between $\vec{L}$ and $\vec{B}$ will remain nearly constant, i.e. $\vec{L}$ will, in effect, follow $\vec{B}$. If, on the other hand, $\vec{B}$ changes direction at a rate that is large compared to $\gamma B$, as in the case where one of its components passes through zero, then $\vec{L}$ cannot follow $\vec{B}$, and the angle between $\vec{L}$ and $\vec{B}$ can change radically. The effect of the field change on the polarization can be deduced from an observation of its effect on the opacity of the vapor. A LabVIEW program called “Optical Pumping Simulation.vi” available on the Junior Lab server can be used on the Junior Lab PC’s to explore these effects (see Appendix A).

5. APPARATUS

Figure 4 is a schematic diagram of the apparatus. Photons with energies in the narrow range in which they can induce $5S_{1/2} \rightarrow 5P_{1/2}$ transitions are obtained from a rubidium vapor lamp excited by a radio frequency generator. The latter subjects the vapor to a rapidly oscillating, high amplitude electric field. The result is an “rf discharge” in which free electrons are driven back and forth with sufficient energy to maintain a state of partial ionization as well as to excite valence electrons to the $5^2 P$ and higher levels.

Some of the photons emitted in the decay of excited states pass through a narrow-band interference filter that transmits only the lower energy photons of the D-line doublet, i.e. the D1 photons produced in $5S_{1/2} \rightarrow 5S_{1/2}$ transitions and having wavelengths close to 7948 Å. The D1 photons have an energy distribution whose width, determined by pressure and Doppler broadening in the lamp, is sufficiently large to encompass the spread in en-
ergy differences between all the $5S_{1/2}$ and $5P_{3/2}$ magnetic substates.

Next the beam passes through a combination of linear polarizer and quarter-wave plate that transmits only photons with one sense of circular polarization, which means that every transmitted photon has a component of angular momentum in the direction of the beam equal to $\pm \hbar$, as the case may be. The beam traverses rubidium vapor and a "buffer" gas (neon) contained at low pressure in a glass bulb.

FIG. 4: Schematic diagram of the optical apparatus and electrical control and display equipment. Collimated light of wavelength 794.8 nm passes through Rubidium gas located at the center of three mutually perpendicular Helmholtz coils. All three components of the magnetic field in the bulb can be controlled. After traversing the cell, the beam is focussed onto a photodiode whose signal is measured with an oscilloscope. When everything is properly adjusted, this configuration can be used to detect very small changes in opacity of the Rubidium Cell.

The rubidium vapor absorption cell is enclosed in a plexiglass oven which allows you to control the vapor pressure of rubidium by adjusting the temperature. The melting point of rubidium is 38.5°C. In addition to rubidium vapor, the cell contains neon gas at a pressure of several mm of mercury. The RF coil is used to irradiate the vapor with very low-energy RF photons to induce transitions between the magnetic substates and thereby depolarize the pumped rubidium atoms. Changes in the intensity of transmitted of the transmitted beam are measured by the solid-state photodiode. The intensity of the signal is displayed on one trace of a dual beam oscilloscope.

NOTE: Electronic equipment and ferromagnetic materials which might have significant effects on the magnetic field should be kept at least a meter away from the vapor cell.

Information on the operation of each component of the setup follows:

5.1. LIGHT SOURCE

The rubidium lamp is a small evacuated glass bulb containing rubidium placed between two coils energized by an RF power supply within the source box. To turn on the lamp:

1. Switch on the lamp power supply;

2. The lamp should reach a stable operating condition (mainly a matter of establishing temperature equilibrium in the lamp) in about 30 minutes. You should turn on the Rb lamp at the very beginning of a lab session due to this length time required for equilibration.

5.2. OPTICS

You will position the two lenses so that the beam is nearly parallel as it traverses the vapor and an image of the light source is focused on the photodiode. There is a 715 nm long-pass filter directly in front of the photodiode which is used to block most of the room lights while transmitting wavelengths above 715 nm. There is also a small plastic shroud used to block overhead light. These two components will allow you to conduct the majority of this experiment with the lights on. They can both be removed (be gentle with them) for the purpose of confirming that the focused beam is squarely hitting the detector.

You will want to be sure that your optical components share a common optical axis which should also be collinear with the center of the rubidium cell (the cell itself is already centered within the Helmholtz coils). This optical axis is 6° and 6.25° above the top of the optical rail for apparatus #1 and apparatus #2 respectively.

Arrange a polarizing filter and a quarter wave plate so as to circularly polarize the beam entering the bulb containing the rubidium vapor. Near the apparatus you will find a supply of linear polarizers, quarter wave plates, and optical bench mounts with which you can construct and test a circular polarizing system. (Avoid touching the optical surfaces with your fingers.) When you are satisfied that you have a combination that works, mount it between the interference filter and lens 1.

5.3. MEASURING CHANGES IN OPACITY

The current induced by light striking the photodiode is amplified by a Spectra-Physics Current Pre-amplier (no. # 70710). This is an in-line current-to-voltage converter with a gain that ranges from $10^4$ to $10^9$. You want to set the gain such that you are getting the highest possible amplification without saturating the system. It is important that changes in the output voltage correspond linearly to changes in light intensity. You can test for
this by checking to see that the voltage responds to both increases and decreases in light intensity.

Next, you will need to keep in mind that the changes in light intensity will be very small (probably on the order of millivolts) relative to the over all level of light intensity (~ 5 to 10 Volts). You can account for this in one of two ways:

- Set the oscilloscope to AC coupling.
- Zooming the scope in as far as possible on the baseline.

5.4. RF GENERATOR (Function Generator 1)

The Agilent 33120A Function Generator will serve as the radio frequency (RF) source. The function generator should be connected directly to the RF coils that are coaxial with the Z-axis. The coils emit RF photons linearly polarized in the Z direction that excite transitions between the magnetic substates of the $5S_{1/2}$ electronic ground state.

The configuration of the function generator will depend on which portion of the experiment you are performing. Specifically, you will use it to sweep through a range of RF, to supply a constant RF, and to possibly emit bursts of RF. An amplitude of 300 mV should work well for generating depolarizing RF.

5.5. HELMHOLTZ COILS

Helmholtz coils are pairs of identical circular coaxial coils separated by a distance equal to their radius and carrying identical currents. They produce a highly uniform magnetic field over an extended region near their geometrical center given by

$$B_I = \frac{\mu_0 R^2 NI}{\sqrt{R^2 + (R/2)^2}} = \frac{8\mu_0 NI}{\sqrt{125}R} \quad (8)$$

where $R$ is the radius of the coils in meters, $I$ is the current through the coils in Amperes, $N$ is the number of turns and $\mu_0 = 4\pi \times 10^{-7}$ Wb A$^{-1}$m$^{-1}$. Remember also that 1 Wb m$^{-2}$=1 Tesla = 10$^4$gauss. The dimensions of the Helmholtz coils used in this experiment are listed in Table I.

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TABLE I: Specifications for both sets of Helmholtz Coils. All dimensions in inches

field which results from not having the optical bench perfectly aligned with the magnetic north-south direction.

3. Longitudinal (Z-axis) Field Coils: As with the X and Y-axis coils, the current through the Z-axis coils is controled through the Helmholtz Coils Power Supplies Panel. However, in the case of the Z-axis coils, there is a switch to toggle the current source for these coils between the adjustable DC power supply and an external source, namely the second Agilent Function Generator. You will use the function generator when a time-varying (saw-tooth or square-wave) current is needed.

The function generator is designed to output to a 50 Ω load. The resistance of the Z-Axis Helmholtz coils will probably only be about 13 Ω. However an additional resistor has been added to the Helmholtz Coils panel to make the total resistance of the external input terminal ~ 50Ω. Yet you will need a way to know how much current is flowing through the Z Coils as a function of time. Thus when using the function generator, you will want to use a BNC T-connector to look at the output of the function generator on the oscilloscope to see what voltage is really being applied to the coils. If you combine this with a measurement of the total resistance of the circuit, you should be able to convert this voltage to a current.

You will use the square wave feature to simultaneously flip the Z-axis field and change its magnitude. The effect this has on the magnetization of the rubidium vapor depends on how the switching time compares with the precession period of the atoms. This may be important in understanding how changes in the magnitude of the X-coil current effects the waveform of the opacity signal. (You can explore the effect of switching time with the program called “Optical Pumping Simulation” on the Junior Lab PC’s - see Appendix A.)
5.6. OVEN HEATER

The vapor cell is in an oven made of plexiglass heated by forced hot air from a blower beneath the table. Turn on the blower, and adjust the Variac for proper temperature. The heating systems on the the two appurtenances are different; each Variac has been marked to indicate what setting will provide a stable oven temperature of 38 \( \pm 4^\circ \) C. Start out with the Variac 20% above this setting for the first ten minutes and then reduce to this setting to maintain the desired temperature. You should turn on the heater at the very beginning of the lab session so that an equilibrium temperature is reached after about 30 minutes.

6. EXPERIMENTS

QUANTITIES YOU CAN MEASURE

1. Separation between the magnetic substates of the ground states of the two natural rubidium isotopes as a function of magnetic field strength.

2. The Landé g-factors and their ratio for the ground states of the two isotopes.

3. \( e/m \)

4. The relative abundance of the two isotopes.

5. The magnitude and direction of the ambient field.

6.1. DEPOLARIZATION BY RF RESONANCE: MEASUREMENT OF THE MAGNETIC MOMENTS OF THE RUBIDIUM ISOTOPES; MEASUREMENT OF THE AMBIENT MAGNETIC FIELD

In this experiment the vapor is continuously optically pumped, resulting in a high degree of polarization of the atomic angular momenta in the gas traversed by the pumping beam. After about one-tenth of a second of pumping, an equilibrium is reached at which the rate of polarization by pumping of the atoms along the beam equals the rate of depolarization by collisions. At that point the vapor becomes relatively transparent because the number of atoms capable of absorbing the circularly polarized photons becomes constant.

6.1.1. Constant Magnetic Field, Varying Radio Frequency

The rate of depolarization can be increased by flooding the vapor with low-energy RF photons tuned to the frequency of the magnetic substate transitions,

\[
f = \frac{\Delta E}{h} = g \mu_B B_z, \tag{9}
\]

where \( \Delta E \) is the difference in energy between adjacent magnetic substates. (The frequency required to induce transitions in \(^{85}\text{Rb} \) \( i = 5/2 \) is approximately 0.5 megahertz per gauss). Photons with the resonant frequency of a particular isotope induce both upward transitions (absorption) and downward transitions (stimulated emission) among the magnetic substates of the ground electronic state of that isotope. The result is an increase in the absorption of the pumping beam.

The resonant frequencies depend on the magnitude of the total field \( |\vec{B}| = (B_x^2 + B_y^2 + B_z^2) \) and not just on its Z-axis component. It is left to you to compute the field produced by the Helmholtz coil from measurements of the coil parameters and the current.

The digital oscilloscope will serve as a high resistance voltmeter with the trigger mode set to “auto”. Estimate at what frequency you expect to see the \(^{85}\text{Rb} \) resonance and then use the knob on the function generator to slowly sweep through this frequency range. If things are set up correctly you should see a noticeable shift in the baseline. Before you attempt this, do you expect the signal level to increase or decrease as you pass through a resonance?

Once you have found the \(^{85}\text{Rb} \) resonance you should be able to quickly calculate (in your head) the frequency at which the \(^{87}\text{Rb} \) resonance will occur. Once you have successfully identified both of these resonances manually, you can program the function generator to perform a sweep through an appropriate frequency range in order to determine the resonant frequencies more precisely. To do this adjust the oscilloscope so that the entire horizontal axis spans a time equal to twice the sweep time (this way you will be able to distinguish real resonant features from noise). Then use the function generator’s SYNC output to trigger the scope.

In both manual and programmed sweeps you will need to watch out for the effects of harmonics of the fundamental oscillator frequency. These harmonic resonances will occur if the amplitude of the frequency generator is set too high.

In this part of the experiment you should aim to obtain the following results:

- magnetic moments of the rubidium atoms in their ground state
- precise determination of the ratio of the Landé g-factors of the two isotopes
- the magnitude and direction of the ambient magnetic field

The following is an outline of a possible procedure:

1. Set the Z, Y, and X-coil currents to zero.
2. Search for the resonance frequencies of the two isotopes in the ambient field.
3. Adjust the X-coil current step-by step so as to minimize the resonant frequency (by bucking out the
vertical component of the ambient field. Tabulate and plot (in your lab book) the X-current and the two resonance frequencies at each step. Trace the hyperbolic relations of frequency versus current on either side of the minima.

4. With the X current set at the value for the minimum resonance frequencies, adjust the Y-coil current in the same manner, and set it to the value for the new minimum resonance frequencies.

5. Finally, tabulate and plot the Z-coil current and resonance frequencies on both sided of the minimum, and see how low a resonance frequency you can achieve (it should be possible to go as low as 10 kHz).

With the data obtained in this way and the coil dimensions, you will have all the information you need to derive the required results. You can use the Digital Magnetometer (with milligauss resolution) to measure and verify your results for the magnitude and direction of the ambient field. It can also be useful for checking that the helmholtz coils are producing the fields you expect.

6.1.2. Constant Radio Frequency, Varying Magnetic Field

By inspection of (9), one can see that the resonance depends on the relation between the magnetic field and the frequency of the RF. So in addition to sweeping through a range of frequencies for a constant magnetic field, we can also detect resonances by linearly varying the magnetic field in a constant RF. Rearranging (9), we see that:

$$B_z = \frac{hf}{g_F\mu_B}$$ (10)

To implement this experimentally, start by setting Function Generator 1 to a constant frequency, say 100 KHz. Use your results from the previous section to buck out the X and Y components of the magnetic field. Then you will want to configure Function Generator 2 with a sawtooth ramp that will cover an appropriate range of the Z-axis magnetic field.

From (10), you can quickly calculate the magnetic field at which resonance will occur for both $^{85}$Rb and $^{87}$Rb. Then, you can determine the current that will be required to generate this field from (8) and measurements of the Z-axis Helmholtz coils.

Then you are left with the task of calculating the voltage range you must specify for the sawtooth on the function generator. To determine this, you will need to measure the resistance of the Helmholtz coils (and the cables that connect it to the function generator). Use a multimeter to measure the total resistance of the Helmholtz coils and associated cables.

Remember that you will need to measure either the voltage across or the current through the coils directly since you cannot trust the values on the function generator.

How many resonant peaks do you expect to see? To answer this, first recall that magnetic substate splitting depends only on the magnitude of the magnetic field and not on the direction. In other words, it does not matter if the resonant value of the field is positive or negative. This would suggest an even number of resonant peaks. However, if you are sweeping through both positive and negative values of the magnetic field, what will happen when the field is very close to zero? What do you expect to happen as you sweep through the value for zero magnetic field (i.e. the current that will produce a field in the Helmholtz coils that will exactly cancel the ambient magnetic field)?

This procedure should provide you with an alternate way of determining the Z-component of the ambient magnetic field.

6.2. Alteration of the Polarization by Changes of the Magnetic Field

Buck out the vertical and lateral components of the earth’s field with the X and Y coils, as above. The Z-coil current can be alternately turned on and off by using a square wave on the the second function generator. The waveforms representing the opacity as a function of time after turn-on and turn-off can be observed and captured on the digital oscilloscope with the opacity signal displayed on one beam, the Z-coil current displayed on the other beam, and the sweep triggered by the SYNC output of the function generator. Precise measurements of the waveforms in the form of amplitude versus time can be made with the horizontal (TIME) and vertical (VOLTAGE) cursors of the digital oscilloscope. (Note that the time constants of the rise and fall of the current in the Helmholtz coil are different.) Record and explain the effects on the waveforms of various changes in the conditions. For example:

1. Change the strength of the field.

2. Change the pumping rate by interposing neutral density filters in the light beam. Several filters of optical density 0.1 are provided. Recall that optical density, $D$, is related to transmission, $T$, by $D = \log_{10} 1/T$

3. Change the density of the rubidium vapor by changing the bulb temperature.

The time constant of the change in opacity following a sudden change in the magnetic field is determined by two competing factors - the intensity of the pumping beam and the rate of disorientation of the atoms through collisions which enhances the “relaxation” and return of the pumped system to its normal state.
You should work out the mathematical theory of the competition between pumping and collisional depolarization in order to interpret your results. (The problem is analogous to that of filling a bucket with a leak in the bottom.)

7. RABI OSCILLATIONS WITH GATED RF PULSES (UNDER DEVELOPMENT)

Another interesting phenomenon can be observed with this apparatus. If you have completed all other parts of this experiment, we invite you to collaborate with us in exploring and understanding a new part (new to us anyway) of this experiment.

When the rubidium cell is feeling only a longitudinal magnetic field and it is suddenly inundated with RF, the opacity signal will exhibit sinusoidal oscillations with a decaying exponential envelope. These are called Rabi Oscillations. The effect looks quite similar to the Free Induction Decay you might have seen in NMR experiment.

To observe this phenomenon, you can use the BURST mode of Function Generator 1. The parameters of the bursts can be set in the MOD menu. You can set the number of cycles pure burst and the frequency at which the bursts will occur.

You might want to start with these parameters:

1. An RF Frequency of 50 kHz
2. 10,000 Burst Cycles

3. A Burst Frequency of 1 Hz
4. an RF Amplitude of 10 V

Once you have these settings in place you will want to look at the photodiode signal at the instant the RF is gated on. A time scale of 500 μs should be appropriate. Once you are seeing these oscillations, the averaging feature on the oscilloscope can make them look quite nice.

You might be feeling that the RF frequency of 50 kHz seems somewhat arbitrary – It is! Notice that this is not a magnetic substate resonant frequency. How do changes in this frequency affect these Rabi Oscillations? How do the oscillations depend on the Z-component of the magnetic field?

8. POSSIBLE THEORETICAL TOPICS

1. Derivation of the Landé-g factors for the two rubidium isotopes, including the effects of the nuclear magnetic moments.
2. Variation of the opacity a) when the field is flipped; b) if the pumping beam were suddenly turned on, and after some time, turned off.
3. Theory of the Helmholtz coils and numerical calculation of the field off axis.

APPENDIX A: SIMULATION OF THE EFFECTS OF FIELD REVERSAL

The magnetic moment \( \vec{m} \) of an ensemble of non-interacting atoms or nuclei under the influence of an external magnetic field \( \vec{B} \) can be described by the equations of motion of an angular momentum \( \vec{L} \) under the influence of the torque \( \vec{m} \times \vec{B} \) where \( \vec{m} = \gamma \vec{L} \), and \( \gamma \) is the gyromagnetic ratio. Thus

\[
\frac{d\vec{L}}{dt} = \vec{m} \times \vec{B} = \gamma \vec{L} \times \vec{B}
\]

(A1)

In a constant field the motion is gyroscoropic precession with a period of \( T = \frac{2\pi}{\gamma B} \). To integrate this equation numerically we express the angular moment at \( t + \Delta t \) in terms of its value at \( t \) by the Taylor expansion

\[
\vec{L}(t + \Delta t) = \vec{L}(t) + \frac{d\vec{L}}{dt}(\Delta t) + \frac{1}{2}\frac{d^2\vec{L}}{dt^2}(\Delta t)^2 + \ldots
\]

(A2)

where each of the terms is evaluated at \( t \), and \( \Delta t \) is adjusted at each step so that it is some fixed and very
small fraction of $T$, i.e. so that $\gamma B \Delta t << 1$. In the optical pumping experiment the magnetic moment is due primarily to the valence electron of the rubidium atom, so the gyromagnetic ratio can be expressed in terms of the charge and mass of the electron by the relation $g = \frac{e}{2mc}$, where $g$, the Landé g-factor, is of the order of unity.

The calculation is coded in a National Instruments LabVIEW program available on the Junior Lab Server. With that program you can compute the motion of the angular moment under your choice of angular momentum orientation (polar angles theta, phi), initial magnetic field $(B_x, B_y, B_z)$, final magnetic field $(B_x, B_y, B_{z1})$, the start and stop times $(tt_1, tt_2)$ of the linear change of $B_z$, and the duration of the computation $(t_{scale} > tt_2)$. An example of the output is shown in Figure 5. Double click on the icon labeled “Optical Pumping Simulation” and follow the instructions.

**APPENDIX B: EQUIPMENT LIST**

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