# ELECTRICAL RESISTIVITY AND HALL EFFECT IN GERMANIUM

(Part II)

REFERENCES

A. C. Melissinos, *Experiments in Modern Physics*, p. 85-98
C. Kittel, *Introduction to Solid State Physics*
F. Reif, *Fundamentals of Statistical and Thermal Physics*

INTRODUCTION

In this experiment the resistivity and Hall effect in a crystal of n-type germanium will be determined as a function of temperature (85 K to 350 K, or $-183°C$ to $80°C$) and the results will be compared to simple and useful theories.

RESISTIVITY AND ITS TEMPERATURE DEPENDENCE:

If a voltage is applied to a bar of solid material, a current will flow through it. The resistance, R, of the crystal is the ratio of the voltage to the current. The resistivity, , is calculated as follows:

$$R = \frac{V}{I} \qquad = \frac{RA}{\ell} \tag{1}$$

$\ell$ is the length of the bar and A is its cross-sectional area.

Germanium is a semi-conductor. The typical behavior of resistivity of a semiconductor versus temperature is shown,
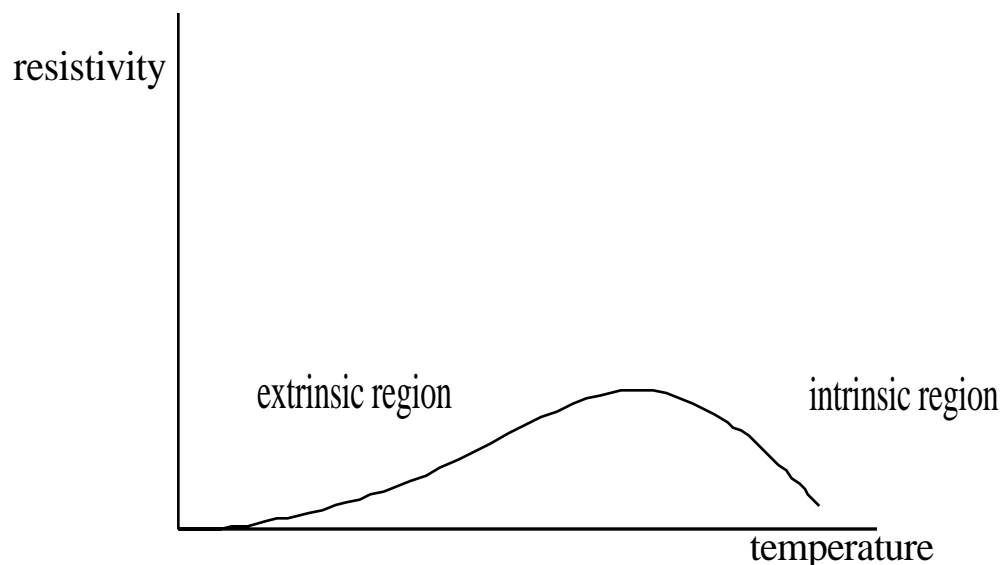
For semiconductors, one must first divide the above figure into low and high temperature regions:

(1)  In the low temperature region, the electrons released from impurities and defects in the crystal conduct the electricity.  This region is called the "extrinsic" region.  The temperature dependence of the "extrinsic" resistivity primarily arises from scattering of these electrons from thermal motion of ions (lattice vibration).

(2)  In the high temperature region, the electrons excited from the valance band to the conduction band for the majority atoms in the semiconductor dominate the conduction of electricity.  This region is called the "intrinsic" region.  In this analysis, we will be concerned with the temperature dependence in both the extrinsic and intrinsic resistivities.

The current density J and the electric field E  in the material bar are

$$J = \frac{I}{A} \qquad\qquad E = \frac{V}{\ell} \tag{2}$$

The resistivity is then expressed as,

$$= \frac{E}{J} \tag{3}$$

Under the influence of the applied electric field E, the electrons that conduct the current travel an *extra* distance    over an averaged time $t$  (the mean flight time) between two successive scattering events,

$$= \frac{1}{2}\frac{eE}{m}t^{2} \tag{4}$$

is also called the mean free path.  The distance traveled with the initial thermal velocity of an electron does not contribute to the total current and is neglected here.  m  is the effective mass of the conduction electrons, which may be different from the mass of free electrons in the case of semiconductors.

We can compute the drift velocity $v_d$ of the electrons as they move through the material bar by

$$v_{d} = \frac{}{t} = \frac{1}{2}\frac{eE}{m}t \tag{5}$$

$v_d$ is much smaller than the mean thermal velocity of the electrons in the n-type germanium.  As a result, the mean flight time between two successive scattering is determined by

$$t = \ell/\bar{v} \tag{6}$$

Since $\bar{v} = \sqrt{3k_B T/m}$ , we have

$$v_d \sim \frac{1}{2}\frac{eE}{m}\frac{\ell}{\sqrt{3kT/m}} = \frac{eE\ell}{2\sqrt{3m\ kT}} \sim \frac{E\ell}{\sqrt{T}} \tag{7}$$

Within the frame work of classical physics, particularly based on the concept of the mean free path for a molecule in a volume of gas, one finds that the mean free path of electrons in a solid $\ell$ is a function of temperature in itself.  As the temperature increases, the positive ions as scattering centers vibrate more. Classically one takes the scattering cross-section to equal the area swept out by a scattering center during its vibration.  This area, $S = \pi r^2$, is proportional to the potential energy of a positive ion and therefore has the form, $V(r) = Kr^2/2$,  K is the effective spring constant.  Since the thermal energy $k_B T$ is equally partitioned between the kinetic energy and the potential energy for a given vibrational mode, we expect the scattering cross-sectional area $S$ to be proportional to T,

$$S = \langle \pi r^2 \rangle \sim \langle V(r) \rangle \sim T \tag{8}$$

The mean free path $\ell$ is related to the cross section $S$ through the equation (Reif, p. 471)

$$\ell = \frac{1}{\sqrt{2}n_s S} \sim \frac{1}{T} \tag{9}$$

where $n_s$ is the number density of positive ions (scattering centers).  Combining equation (7) and (9), we have

$$v_d \sim \frac{E}{T^{\frac{3}{2}}} \tag{10}$$

If there are $n_e$ electrons per unit volume, the electrical current density $\vec{J}$ is given by,

$$\vec{J} = n_e(-e)\vec{v}_d \sim \frac{E}{T^{\frac{3}{2}}} \qquad \textbf{(extrinsic region)} \tag{11}$$

Then    should have the following temperature dependence

$$= \frac{E}{J} \quad T^{\frac{3}{2}} \qquad \textbf{(extrinsic region)} \qquad (12)$$

Such a simple classical theory qualitatively describes the temperature dependence of the resistivity for germanium in the **extrinsic region** when the electrons density is roughly a constant.  In the **intrinsic region** (read the materials on *Experiments in Modern Physics by* Melissinos from page 80 to 98),  the electron density is dominated by that of electrons thermally excited from the valence band to the conduction band of germanium,

$$n_i \quad \left( \frac{2 \ m \ k_B T}{h^2} \right)^{\frac{3}{2}} \exp \left( -\frac{E_g}{2k_B T} \right) \qquad \textbf{(intrinsic region)} \qquad (13)$$

$$\vec{J} = n_i(-e)\vec{v}_d \quad \exp \left( -\frac{E_g}{2k_B T} \right) \qquad \textbf{(intrinsic region)} \qquad (14)$$

$$= \frac{E}{J} \quad \exp \frac{E_g}{2k_B T} \qquad \textbf{(intrinsic region)} \qquad (15)$$

A related concept is the drift mobility $\mu$ which is the ratio of the magnitude of the drift velocity to the applied electric field

$$\mu = \frac{\vec{v}_d}{\vec{E}} = \frac{\vec{J}}{\vec{E}} \frac{1}{n_e(-e)} = \frac{-1}{(n_e e)} \quad T^{-\frac{3}{2}} \qquad (16)$$

This analysis assumes that the electric current is carried by electrons only. For information on the treatment of two types of charge carriers, see p. 86 of *Experiments in Modern Physics by* Melissinos.

## HALL EFFECT

When an electric current $\vec{J} = n_e(-e)\vec{v}_d = J\hat{y}$, driven by an applied electric field $\vec{E} = E\hat{y}$, is flowing along y-axis (J > 0) in a bar of solid material placed in a magnetic field $\vec{B} = B\hat{z}$ along z-axis, the conducting electrons experience a magnetic force

$$\vec{F}_m = -e\,\vec{v}_d \times \vec{B} \tag{17}$$

The force is perpendicular to the current flow ($\vec{v}_d$), and causes the accumulation of electric charges on the two faces of the bar that are parallel to the directions of the current and the magnetic field. In this case, the excess electrons are accumulated on the surface at x = 0, and an equal amount of excessive positive charge is accumulated on the surface at x = - W.
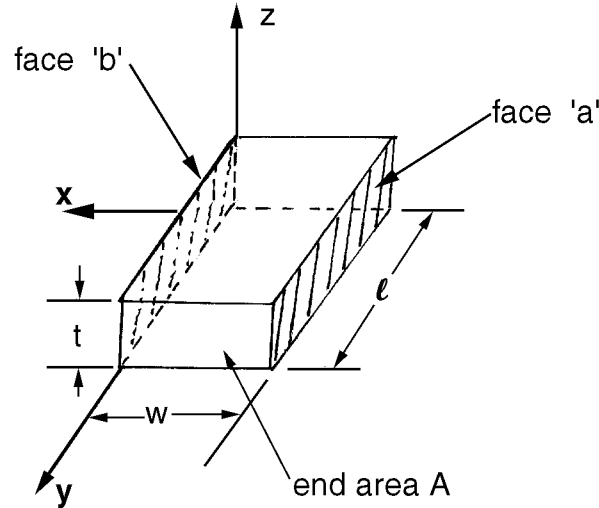


The accumulated electric charges on the shaded faces produces an electric field pointing along the positive x-axis and a voltage difference between the shaded faces develops. The electric field along the x-axis produces an electrostatic force that eventually balances the magnetic force. This resultant electric field $\vec{E}_x = E_H\hat{x}$ is the Hall field $\vec{E}_H$, and the associated voltage difference is the Hall voltage $V_H$. Clearly along x-axis,

$$\vec{F}_E + \vec{F}_m = (-e)(\vec{E}_H + \vec{v}_d \times \vec{B}) = 0 \tag{18}$$

and thus

$$\vec{E}_H + \vec{v}_d \times \vec{B} = 0 \tag{19}$$

Thus for electrons with $\vec{v}_d = v_d \hat{y}$ ($v_d < 0$) pointing in the direction opposite to $\vec{J}$,

$$E_H = -v_d B \tag{20}$$

In term of mobility $\mu = \vec{v}_d / \vec{E} = v_d / E$, we can express the Hall field as

$$E_H = -\mu E B \tag{21}$$

If we define the Hall coefficient $R_H$ as

$$R_H = \frac{E_x}{J_y B_z} = \frac{E_H}{JB} \tag{22}$$

by combining equation (21) and (22), we find

$$R_H = -\mu \quad ; \tag{23}$$

by combining equation (11) and equation (20) with equation (22), we arrive at

$$R_H = -\frac{v_d}{J} = \frac{1}{n_e e} > 0 \tag{24}$$

If the electric current is dominated by holes that carry positive charges, we will find equation (20) and equation (24) are still valid with $v_d > 0$. Namely, $R_H < 0$. This means that given the directions of the electric current and the applied magnetic field, one can determine the sign of the electric charge carriers from the direction of the Hall field or equivalently the sign of the Hall coefficient $R_H$.

In the intrinsic region of a semiconductor, both electrons and holes participate the electric current, the suitable equation for the Hall coefficient can be found on page 87 of *Experiments in Modern Physics by* Melissinos. It can be empirically written as

$$R_H = \frac{r_H}{n_e e} \tag{25}$$

with $r_H$ close to unity if one type of charge carriers dominates the electric current.

EXPERIMENTAL PROCEDURES:

The resistivity and Hall effect measurements can be done simultaneously. The data are collected over a temperature range from $-180°C$ to $80°C$. The apparatus is shown below.
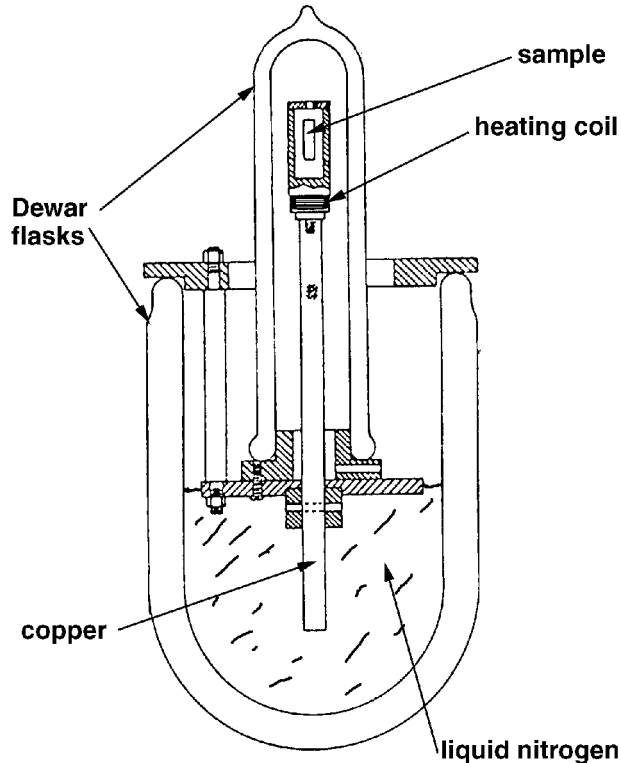


The germanium single crystal sample is enclosed in a copper chamber for the purposes of both temperature regulation and protection. It has a width $w = 5$ mm, a thickness $t = 1.1$ mm, and a length $\ell = 13$ mm. **PLEASE DO NOT OPEN THE SAMPLE CHAMBER**.

The sample crystal is wired for a four-lead resistance measurement as shown in the next page. It allows the voltage across the sample to be measured without the interference of the contact resistance. A high impedance voltmeter is used so that an almost negligible current flows through the connecting wires.

**The total current through the germanium crystal should be less than 4 mA.** To provide this limitation, a current-limiting resistor of 10 K is connected in series with the germanium sample which has a nominal resistance of 300 . A dc power supply is used to apply a voltage of 20 to 30 volts across the resistor-sample network. **Make sure that the applied voltage does not exceed 40 volts !!**

The sample and the wire connection have the following geometry:
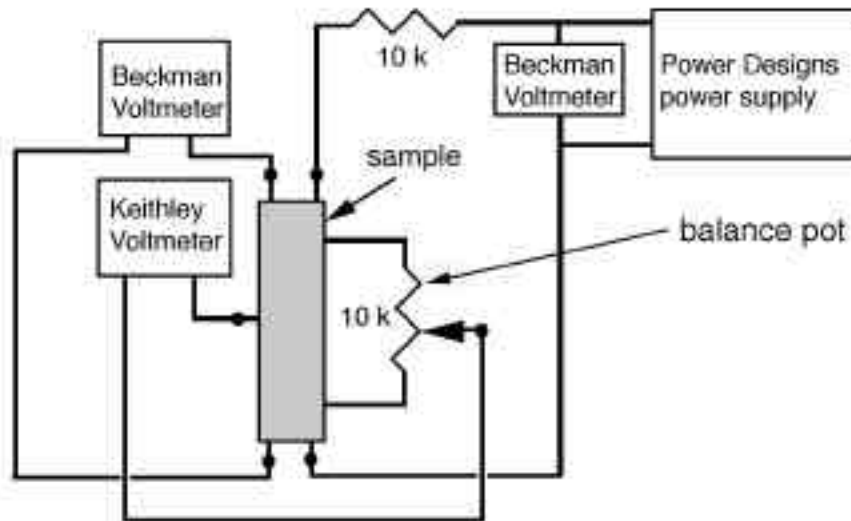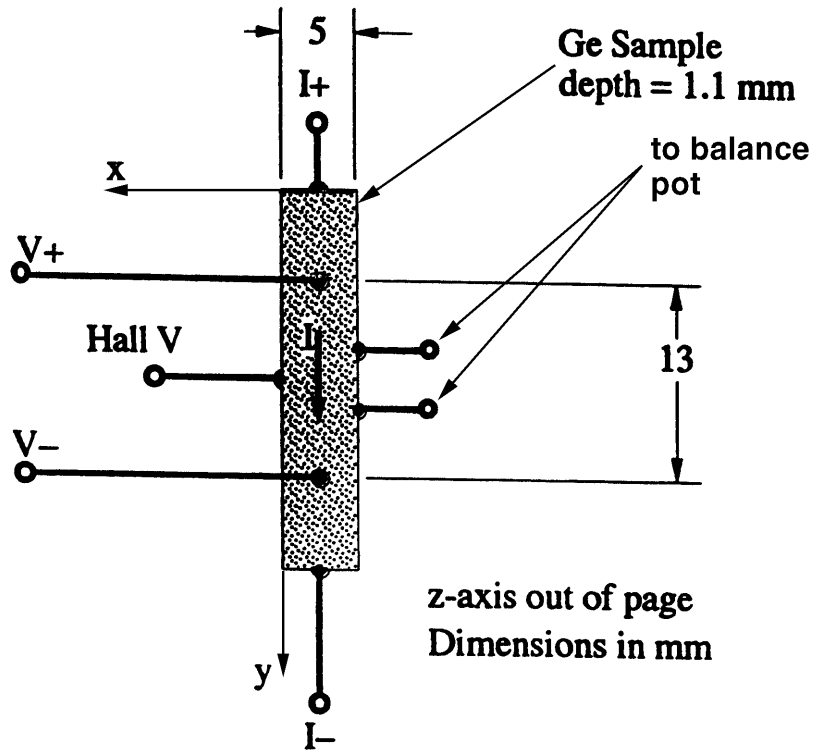




To measure the Hall voltage $V_H$, one can not simply attach two leads on the two opposite surfaces of the sample that are parallel to the current and the applied magnetic field. This is because that a slight misalignment of the two leads along the direction of the current would introduce an ohmic voltage drop along the direction of the current to the "measured value" of the Hall voltage.

To eliminate such an ohmic voltage in the Hall voltage measurement, an adjustable potentiometer (the balance pot) is added between two spaced-out contact points on one side surface of the sample. A potential difference exists between these two points due to the ohmic potential drop along the current path when the magnetic field is removed. There is a point along the current path in the balance pot which is at the same potential as the contact point on the opposite side in the absence of the applied magnetic field. You measure the Hall voltage between this point and the contact point on the opposite surface in the presence of magnetic field. Adjust the potentiometer to null this voltage in the absence of a magnetic field.

The horse-shoe shaped magnet of field strength of roughly 800 gauss is placed with the small Dewar flask in the gap. Carefully rotate the magnet about a vertical axis until the maximum in the Hall voltage is achieved. This is the position that the magnetic field is perpendicular to the 5-mm $\times$ 13-mm surfaces.

TEMPERATURE REGULATION OF THE CRYOSTAT

The control of the sample temperature is achieved through the liquid nitrogen cooling and the electrical heating. The sample is attached to one end of an isolated copper rod while the other end is submerged in a liquid nitrogen dewar ($LN_2$). Close the sample end, the copper rod is wound with many turns of electrical heating wires. The combination of the electrical heating and the $LN_2$ cooling provide a desirable sample temperature range from $-181^{\circ}C$ to $80^{\circ}C$.

A thermocouple sensor is placed next to the sample. With the reference probe submerged in ice water, the thermocouple readings is fed to a Eurotherm 808 temperature controller. The controller regulates the sample temperature by controlling the electrical heating power to the copper rod, based on the difference between the actual sample temperature and the user set temperature.

Here is the procedure to use the Eurotherm 808 controller to control the sample temperature:

(1) Make sure that the sample has been cooled with liquid nitrogen for a couple of hours. You want to fill the liquid nitrogen (to the blue line on the inside wall of the dewar to avoid submerging the apparatus other than the copper rod) before the class start to save time;
(2) Make sure that the reference probe is submerged in ice water;
(3) Make sure that the banana plugs for the thermocouple from the sample assembly is connected to the temperature control box;
(4) Make sure that the four-pin connector for delivering the electrical heating power is temporarily disconnected;
(4) Turn the switch next to "AC" on the control box to "on";

(5) The front panel of the Eurotherm 808 Controller should have two displays: the upper display is the current sample temperature, obtained from the thermocouple; the lower display is the set temperature;
Since the heating power is cut off from the sample assembly, the difference between the set temperature and the sample temperature remains;

(6) You can change the set temperature by pressing the up arrow " " or down arrow " " on the front panel;
Initially, set the temperature at or below the current sample temperature;
**(DO NOT SET THE SET POINT OVER 100 °C !!)**

(7) Connect the four-pin connector is connected from the sample assembly to the temperature box;
(Note: make sure that the orientation of the four pins is correct as indicated by the matching bumps on the connector and on the receptacle);
There should be no change in either the set temperature and the sample temperature

(8) Now increase the set temperature to a desired value, you will notice that the sample begins to increase;
The temperature rise continues until the two temperatures are equal.
(Note: the clicking sound comes from the relay inside the control box being activated or deactivated, a sign of healthy temperature regulation).
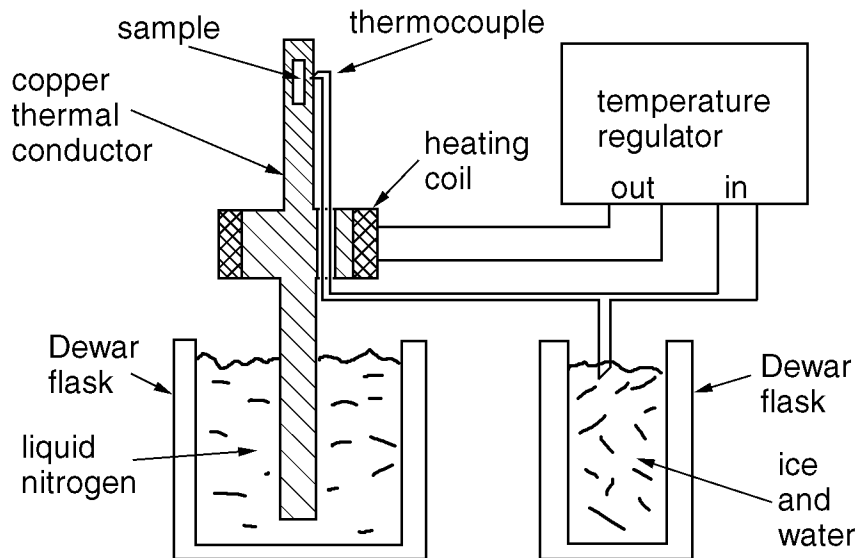
<u>DATA ANALYSIS FOR RESISTIVITY MEASUREMENT:</u>

In the **extrinsic region**, namely, at temperatures well below the maximum of the resistivity, the resistivity is expected to have a power law or $T^a$-dependence on temperature as shown in equation (12). Calculate $a$ from your data.
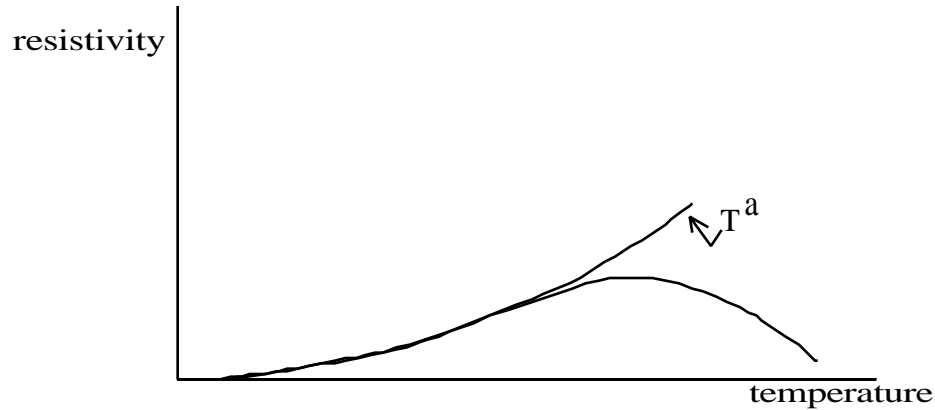


When a set of data follow a power law, such as that described by equation (12), the proper display of the data is a log-log plot. For example, if one has the equation

$$\rho = c\, T^a \tag{26}$$

By taking logarithms on both sides, one obtains,

$$\ln \rho = \ln c + a\, \ln T \tag{27}$$

which is the equation of a straight line with slope $a$. and intercept $\ln c$. One can then make a least squares fit to equation (27) to find the best value of the exponent $a$.

In the **intrinsic region**, namely, at temperatures above the maximum of the resistivity, the resistivity is expected to have an Arrhenius $\exp(a/T)$-dependence on temperature as shown in equation (15). Determine the energy gap $E_g$ from your data.

When a set of data follows an Arrhenius law as is $\exp(a/T)$, the proper display of the data is an Arrhenius plot. For example, if one has the equation

$$\rho = c\exp(a/T) \tag{28}$$

By taking the logarithm on both side, one obtains

$$\ln \ = \ \ln c \ + a/T \tag{29}$$

An Arrhenius plot of   is a plot of ln   versus $1/T$. It is a straight line again with a slope a and the intercept ln c. One can make a least-square fit to equation (29) using   as the data to find $E_g/2k_B$ and in turn $E_g$.


## DATA ANALYSIS FOR HALL EFFECT MEASUREMENT

You will measure $R_H$ as a function of temperature. From these data and the resistivity data, compute $\mu$ and $r_H/n_e$ versus temperature.